A Short-Term Operational Planning Model for Natural Gas Production Systems

Ajay Selot
Dept. of Chemical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139

Loi Kwong Kuok and Mark Robinson
Smart Fields Technology, Sarawak Shell Berhad, Miri, Sarawak, Malaysia

Thomas L. Mason
Shell International Exploration and Production B.V., Rijswijk, The Netherlands

Paul I. Barton
Dept. of Chemical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139

DOI 10.1002/aic.11385
Published online December 21, 2007 in Wiley InterScience (www.interscience.wiley.com).

A short-term operational planning model for natural gas production systems can help identify a consistent operational policy that satisfies contractual rules and customer specifications. Formulating and solving such a model poses challenges due to nonlinear pressure-flowrate relations, a multicommodity network and complex production-sharing contracts (PSC). A production allocation model is presented that can be viewed as a contractual model superimposed on an infrastructure model. The infrastructure model incorporates nonlinear pressure-flowrate relationships for wells and pipelines, multiple qualities of gas in the trunkline network and models of facilities. The contractual model is a mathematical representation of the PSC and associated operational rules. The model features are inspired by the Sarawak Gas Production System (SGPS) in East Malaysia. A case study similar to the Sarawak Gas Production System (SGPS) is presented. The final formulation is a nonconvex mixed-integer nonlinear program and is solved with GAMS/BARON to global optimality. A hierarchical multiobjective operational study is also presented. *AIChE J,* 54: 495–515, 2008
Keywords: natural gas systems, natural gas supply chain, contracts model, operational planning, production planning

Introduction

Natural gas has become an increasingly important fuel in recent years. World natural gas demand is expected to almost double by 2030 and by then, natural gas is forecasted to overtake oil as the dominant fuel in the industrial sector. Worldwide natural gas consumption is forecasted to grow at an average rate of 2.4% annually as compared with 1.4% for oil in the next 25 years.[1]

This article contains supplementary material available via the Internet at http://www.interscience.wiley.com/jpages/0001-1541/suppmat.
Smart Fields is a registered trademark of Shell.
Correspondence concerning this article should be addressed to P. I. Barton at pib@mit.edu.

This demand is primarily driven by industrial users and electricity generation. Natural gas is a less carbon-intense fuel than either oil or coal, i.e., its combustion produces less greenhouse emissions per unit energy produced. Moreover, it produces relatively lower sulfur, NOx, and particulates emissions on combustion compared to other fossil fuels. In big emerging economies such as China and India, concerns over pollution are expected to drive natural gas demand, especially LNG, for use in electricity generation. Another big area of growth driven by high oil prices is the conversion of natural gas to liquid fuels (gas-to-liquids, GTL), an economically attractive option for exploiting remote fields.[1]

Natural gas is inherently difficult to store and transport. It requires an extensive pipeline and compression infrastructure for transport, and mostly natural geological formations for storage. Alternatively, for transport and storage as LNG, LNG plants for liquefaction, LNG terminals for loading, offloading and regasification and LNG tankers for transportation are required. These difficulties have led to the practice of long-term contractual agreements between producers and consumers in the natural gas industry. This has helped to mitigate risks at the expense of making routine operational decisions harder. As production systems develop over time and new fields come online, new contracts and rules add to the already existing contractual arrangements. The problem is exacerbated by a multitude of parties involved in the development of the fields and also by the involvement of national governments (or government controlled national oil and gas companies), giving rise to complicated production-sharing contracts. In the end these rules become too complicated to formulate a production policy by mere inspection or by merely optimizing a model of the physical production system. Also, upstream natural gas production systems handle final products directly as opposed to oil production where product separation occurs at a refinery. This implies that customer specifications and production-sharing contracts (PSC) directly affect the operation of the system and any operational planning model has to take them into account to be useful.

Most of the previous work on optimization of natural gas production systems has focused on two aspects: long-term planning and development of offshore infrastructure, and short-term planning for local distribution companies (LDC). The upstream short-term problem with contractual rules seems to have been overlooked, partly because computing power and optimization theory have been insufficient in the past to handle the kind of relatively large-scale nonconvex mixed-integer nonlinear programs that result from these production planning problems. However, we believe that with the advances in global optimization theory and algorithms made in the last decade, this problem is now tractable, provided a proper optimization modeling approach is used in conjunction with exploitation of problem structure.

In this work, the Sarawak Gas Production System (SGPS) in Malaysia is used to illustrate the issues associated with short-term operational planning in upstream production systems. This is the first attempt, to the best of our knowledge, to model and optimize the complex production-sharing contracts coupled with a technical model of the upstream natural gas supply chain and apply it to a real world system.

The article is organized as follows. Relevant previous works are discussed. Since the model is inspired by the SGPS, an introduction to the SGPS is presented to provide motivation and requirements for an operational planning model. A general modeling methodology for upstream natural gas systems is presented as a contractual model constraining an infrastructure model. A case study similar in complexity to the SGPS is discussed. Finally, a hierarchical multiobjective study is presented. The Appendix presents a brief description of the contractual rules modeled in the case study. Supplementary material to the article provides additional information on the case study.

Literature Review

The oil and natural gas industry has been a pioneer in the application of mathematical programming with work going back to at least 1952.[2] Therefore the body of literature on this topic is vast and an exhaustive survey is outside the scope of this paper. Our main focus here is work that either directly concerns natural gas subsystems modeling or is relevant to the modeling approach presented.

Dougherty[3] presents a review of work in the field until 1970 from a petroleum engineering perspective that covers both oil and natural gas applications. Another survey of the work prior to 1977 can be found in Durrer and Slater.[4] Broadly, the work relevant to natural gas modeling can be divided into the following topics:

(1) Infrastructure development planning, both in oil and natural gas fields.

(2) Decision support models for LDC or utility companies for planning purchase, storage and transportation of natural gas.

(3) Short-term gas transmission system modeling, simulation and optimization.

(4) Short-term production planning models for oil fields.

Offshore infrastructure development and long-term planning is the earliest and possibly the most widespread application in this area. This is not surprising given the enormous capital cost and risk associated with offshore field development. However, because this work is concerned with operational planning, only some representative works and trends in the area are outlined and one should refer to these to explore the field further. A combination of the infrastructure and operational problems from a petroleum engineering perspective appears in Huppler,[5] Flanigan[6] and O'Dell et al.[7] A well location problem with a simplified reservoir model is solved in Murray and Edgar.[8] McFarland et al.[9] use a simple tank model for reservoir dynamics to formulate an optimal control problem and solve it using a generalized reduced gradient method. A long-term multiperiod mixed-integer linear program (MILP) model in use by the Norwegian regulator to aid in field development decision making is presented in Nygreen et al.[10] van den Heever et al.[11] present a model for long-term infrastructure planning with complex economic objectives. A model of the production system in Saudi Arabia and issues associated with it is presented by Gao et al.[12] An oil well spacing and production control problem is solved in Ayda-zade and Bagirov[13] by first formulating a two dimensional partial differential equation (PDE) model of the reservoir and then converting it to a conventional optimization problem. Recently, work has started focusing on handling the uncertainty involved in planning using stochastic programming formulations.[14,15] A simple analysis of the profitability of development projects in the presence of production-sharing contracts appear in Yusgiantoro and Hsiao.[16] The analysis does not involve mathematical programming and is based on simply evaluating several scenarios.

A substantial body of work exists on operational optimization of gas pipeline transmission systems, the main objective being to minimize the operational costs for the network. An optimal control perspective on the problem can be found in Marqués and Morari[17] and Osiadacz and Bell.[18] Furey[19] presents a modified successive quadratic programming (SQP) algorithm for optimizing natural gas pipeline networks. A bundle method is used to solve the pipeline design problem in De Wolf and Smeers.[20] In a later work by the same authors,[21] an iterative method using piecewise linearizations of nonlinear functions and LP simplex is employed to solve the network problem. An optimal routing problem for natural gas transportation is presented in Dahl et al.[22] A reduction method for networks is presented in Ríos-Mercado et al.[23] Techniques for constructing piecewise linear approximations of the nonlinear functions involved in the gas transmission network and the properties and solution of the resulting MILP have recently appeared in Martin et al.[24] A methodology based on dynamic programming for minimizing fuel consumption in gas networks that contain cycles is presented in Ríos-Mercado et al.[25]

Substantial effort has been put into decision support models for making purchasing and storage decisions for LDC and gas transportation companies. An optimal schedule for withdrawal from storage reservoirs appears in Wattenbarger.[26] An allocation problem within a statewide trunkline network with users having different priorities is solved in O'Neill[27] that introduces the idea of a pseudonetwork to model swaps between different pipeline systems. Levary and Dean[28] present a model for gas procurement by a natural gas utility. A LP framework to evaluate supply scenarios for planners on a statewide or national level is presented in Brooks.[29] A chance-constrained approach to making purchasing and storage decisions for a utility is presented in Guldmann.[30] Another LP model for determining utility decisions appears in Avery et al.[31] A model for a Chilean LDC with contracts is presented in Contesse et al.[32] Recently Gabriel et al.[33] present a mixed nonlinear complementarity model of natural gas markets.

Work has also focused on short-term production planning in oil fields. Kosmidis et al.[34] present a model for well rate allocation that comprises naturally flowing and gas lift wells. The model is solved by linearizing well models to formulate an approximate MINLP and then solving a sequence of MILPs. Ortíz-Gómez et al.[35] present MILP and MINLP short-term multiperiod oil production models, however nonconvex MINLP models are not solved to global optimality and they only address oil production systems where the gathering system is not strongly coupled with the wells. Barragán-Hernández et al.[36] solve a dynamic model with an interior point solver for an oil field.

A detailed discussion of issues involved in natural gas supply chain planning appears in Tomasgard et al.[37] The Energy Information Administration of the US Department of Energy has developed over years a demand, supply and transportation matching model for the North American natural gas market.[38]

Mixed-Integer Nonlinear Programs

A general mixed-integer nonlinear programming problem (MINLP) can be represented as:

$$\begin{aligned} \min_{\mathbf{x},\mathbf{y}} \quad & f(\mathbf{x},\mathbf{y}) \\ \text{subject to} \quad & \mathbf{g}(\mathbf{x},\mathbf{y}) \leq 0 \\ & \mathbf{h}(\mathbf{x},\mathbf{y}) = 0 \\ & \mathbf{x} \in \mathbb{X}, \quad \mathbf{y} \in \{0,1\}^{n_y} \end{aligned}$$

where $\mathbb{X} \subset \mathbb{R}^{n_x}$ is a nonempty convex set, usually an interval. $f : \mathbb{X} \times [0,1]^{n_y} \longrightarrow \mathbb{R}$, $g_i : \mathbb{X} \times [0,1]^{n_y} \longrightarrow \mathbb{R}$ and $h_i : \mathbb{X} \times [0,1]^{n_y} \longrightarrow \mathbb{R}$ are continuous functions, some or all of which are nonconvex. If the integer variable vector $\mathbf{y}$ is relaxed to be in the interval $[0,1]^{n_y}$, a nonconvex nonlinear program is obtained. It is worth noting here that nonlinearity of equality constraints h_i (even though they may be convex) is sufficient to make the nonlinear program nonconvex, in almost all cases. MINLPs in which some or all the participating function are nonconvex will be referred to as nonconvex MINLPs.

Deterministic optimization algorithms can be characterized on the basis of the nature of their solution set as being global optimization algorithms and local optimization algorithms. Local optimization methods are called so, since they are only guaranteed to converge to a local minimum (more precisely, to a point satisfying some necessary condition for a local minimum, e.g., Karush-Kuhn-Tucker (KKT) conditions[39,40]). Almost all nonlinear programming algorithms (e.g., sequential quadratic programming (SQP)) fall into this category. Local algorithms are unsuitable for nonconvex programming for several reasons, most important being that nonconvex programs usually have multiple local minima some of which can be suboptimal and KKT conditions are not sufficient to characterize a global minimum for nonconvex programs. General theoretical difficulties aside, many local NLP algorithms (also referred to as local solvers later) behave erratically for nonconvex programs since several assumptions specific to the algorithms and their implementation break down for these programs. In the worst case, they can be misleading, reporting a problem whose feasible set is non-empty as being infeasible. More information on general nonlinear programming theory can be found in Bertsekas[39] and Bazaraa et al.[40]

Hence, global optimization algorithms that can guarantee convergence to a global minimum are required for reliable solution of continuous nonconvex programs and by similar extension of logic for nonconvex MINLPs. Most global optimization algorithms are not specific algorithms, but are algorithmic frameworks within which details should be filled in to arrive at a specific algorithm. In general, these algorithms work by solving a series of subproblems using local solvers to generate upper and lower bounds on the optimal solution value. These upper and local bounds will converge to within the specified accuracy after a finite number of iterations under mild assumptions on the nature of the subproblems. The robustness of global optimization algorithms stems from their exhaustive search over the feasible set of the problem,

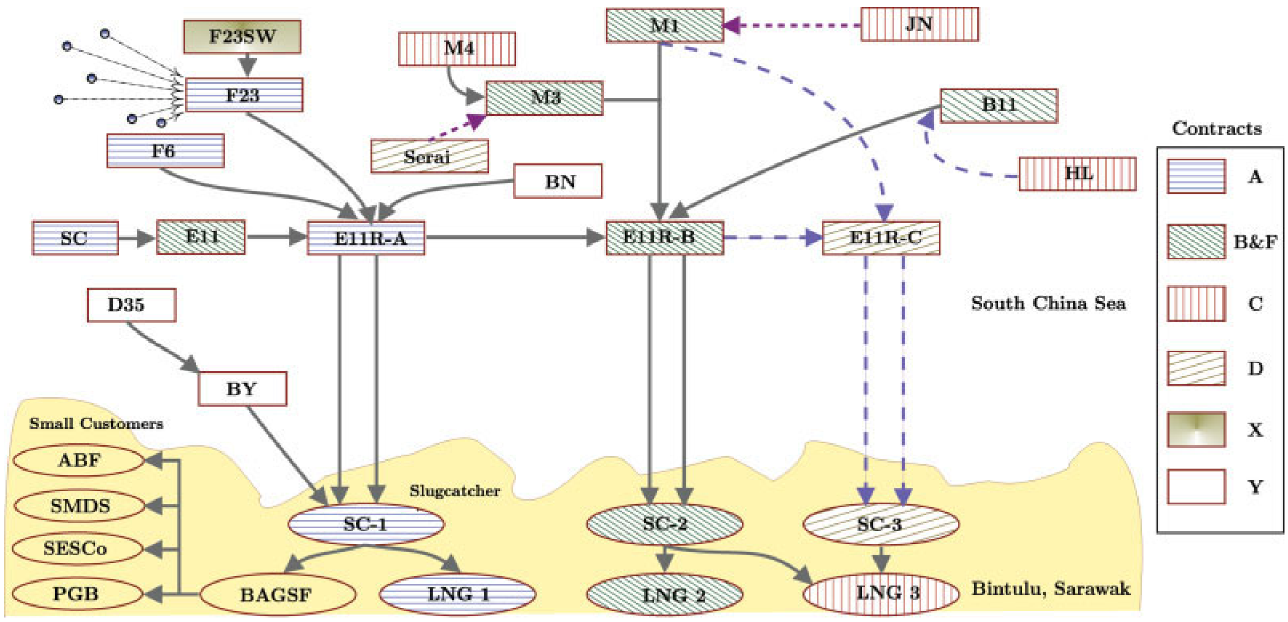

Figure 1. Sarawak gas production system: network overview (not to scale).

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

infeasibility decisions relying on overestimation (relaxation) of the feasible set and their globally convergent nature (convergence is independent of the initial guess for starting the algorithm). However, global optimization algorithms have worst-case exponential runtime and hence solving even moderate-sized nonconvex MINLPs (with several hundred continuous variables and tens of binary variables) to global optimality can be quite challenging. Hence, in general, global optimization algorithms need to be customized, i.e., the general frameworks need to be tailored to a specific problem class using features of the class to accelerate convergence.

A problem can have several MINLP representations that are equivalent, i.e., their optimal solution set and optimal solution value are the same. However, these representations may not be equivalent in terms of the effort required to solve them. Different equivalent MINLP representations result in different subproblems that must be solved at an iteration of a global algorithm, thus strongly affecting its convergence. Hence, attention to modeling is of paramount importance for nonconvex MINLP and is as crucial as the algorithms employed to solve the model. Further information on MINLP solution techniques can be found in the review by Grossmann.[41]

The Sarawak Gas Production System

The SGPS is located in the South China Sea off the coast of the state of Sarawak in East Malaysia. There are 12 offshore gas fields in the system. Additionally, associated gas from 3 oil fields is fed into the system. The daily production rate of dry gas from the SGPS is around 120 million standard m^3/day. Additionally the system also produces 15,000 m^3/day of natural gas liquids (NGL). The annual revenue from the SGPS is around $5 billion, that is approximately 4% of Malaysia's GDP.

From a modeling standpoint, the system comprises wells in the fields, well platforms, the pipeline network and the facilities onshore. There are both sweet fields and sour fields in the system. Gas from sour fields contains a relatively large amount of H_2S and CO_2, while gas from sweet fields is mostly free of these contaminants.

Gas from the wells belonging to a particular field is collected at a well platform. A well platform may serve more than one field. Well platforms have dehydration facilities that perform a three phase separation of gas, NGL, and water. They may also have compression facilities, in case the field pressure is insufficient to drive the flow. Once dehydrated, dry gas and NGL are remixed (after compression if a platform has compression facilities) before injection into the pipeline. A subsea pipeline network (referred to as the trunkline network later) connects the well platforms to the facilities onshore.

The subsea trunklines end at one of the three slugcatchers corresponding to the three liquefied natural gas (LNG) plants at the complex in Bintulu, Sarawak. The slugcatchers are units that remove NGL from the two phase flow coming out of the trunklines. The liquids are sent to stabilizers to remove volatiles and the dry gas is fed into the LNG plants. The LNG plants supply primarily Japan and South Korea. There are also small customers: a power generation company, a fertilizer plant, a local utility and a petrochemical plant. However, these users consume close to 5% of the total production and moreover their demand is mostly fixed and hence can be represented by adding a small constant factor to the minimum demand rate of the first LNG plant. Hence these need not be considered explicitly for planning. The complex also contains a liquefied petroleum gas (LPG) plant. Hence, there are two by-products, NGL and LPG, from the system. A network overview of the SGPS is presented in Figure 1.

Operational aspects

The fields, facilities and plants in the system are not owned by a single entity but instead several parties either have stakes in them or may fully own some of them. However, almost the entire system (excluding the customer plants) is operated by a single upstream operator. As a consequence of ownership issues, a particular field cannot arbitrarily supply any customer in the system. There are production-sharing contracts (PSC, also referred to simply as contracts later) and sales agreements that determine how the products are to shared between different parties and their specifications. In particular, these specify field-plant assignments, transfers between different contracts, operational rules, gas quality specifications, and facilities usage rules. The SGPS operation is therefore governed by an extensive set of rules that must be satisfied at all times. Hence, making routine operational decisions about production and routing in the network is difficult.

Traditionally this has been done in two stages: first solving a production planning problem using the production system model with a local solver (e.g., SQP) to determine a feasible set of well production rates and trunkline flowrate-pressure distribution, and then manually checking if the rules are satisfied. If not, then another scenario can be evaluated by enforcing different constraints and bounds on the production system model and checking the rules again. Iteratively, a feasible solution that satisfies the rules may be found. However, this approach suffers from several problems:

(1) Because of the nonlinear pressure-flowrate relationships in the wells and trunklines, the problem is nonconvex. Hence solution of the first stage production planning problem is liable to fail. Our numerical experiments show that local solvers may report some feasible instances of the problem as infeasible.

(2) Even if this iterative procedure does converge, there is no guarantee concerning the quality of the solution, i.e., there is no information if a far superior solution exists.

(3) For a large system containing tens of fields, such a scheme is too tedious and error prone to devise a consistent operating strategy.

(4) It is also possible that this procedure may not generate any solution point at all that satisfies all the rules.

Table 1. Common Symbols

Symbol	Description	Units	Type
$P_{(.)}$	Pressures	MPa	Variable
$Q^{(.)}_{(.)}$	Dry gas volumetric rates in infrastructure model	hm^3/day*	Variable
$Q^{(.)}_{L(.)}$	NGL volumetric rates in infrastructure model	m^3/day	Variable
$q^{(.)}_{(.)}$	Volumetric rates in contract network	hm^3/day	Variable
$F^{(.)}_{(.)}$	Molar rates	Mmoles/day	Variable
$y^{(.)}_{(.)}$	Binary variables		Variable
$\pi_{(.)}$	Pressures	MPa	Constant
$\theta_{(.)}$	Temperatures	K	Constant
$\mathcal{A},\mathcal{B},\mathcal{C}...$	Sets		Set symbol
(i, j)	A directed arc from node i to j		Index
i, j, k	Indices		Index

*hm^3 = 10^6 m^3 (since 1 hectometer = 10^2 m).

The Upstream Natural Gas Production Allocation Model

This work focuses on short-term production allocation in the upstream system. By short-term, planning on the order of a few days to a few weeks is implied. The upstream system is defined as from the bottom of the well bore to the LNG plants, however excluding the LNG plants. The operating state of the system is determined by the following decision variables: Production share of dry gas from each well (and therefore each field), associated pressures at the well bore and well head for each well, pressure and flowrate distribution in the trunkline network, the state of inter-contract transfers and operational rules and, amount and quality of gas delivered to the LNG plants.

Model requirements

The objectives of the problem are with respect to the perspective of the upstream operator managing the production system. Hence, the goal is to formulate an operating policy such that the operator can optimize its objectives while simultaneously satisfying all the rules. The model is supposed to be a decision support tool for operators controlling the system and to help them plan a steady-state operation between disruptions (e.g., a field needs to be temporarily shutdown due to a breakdown or a facility needs emergency repairs) or planned events (e.g., a scheduled maintenance shutdown). Therefore, a multiperiod formulation is unnecessary for this problem. Also, only operational objectives are considered since they are adequate for the purposes outlined above.

Following are the requirements for the production planning model and a discussion of some model features resulting from these requirements:

(1) The entire network is controlled by regulating pressure at the slugcatchers. Hence it is essential to model reasonably the nonlinear pressure-flowrate relationships in the trunkline network and in the wells.

(2) There are different qualities of gas (i.e., gas with different composition) in the network and hence species flowrates need to be tracked throughout the network.

(3) The model needs to include customer specifications on gas quality.

(4) The contractual rules need to be included in this model.

(5) A requirement for the model is that it be extensible so that more detailed models for facilities can be added later.

From these requirements, it is clear that the final model will be a nonconvex MINLP.

Model overview

It is instructive to view the overall model as being the two following submodels that are coupled:

(1) *The Infrastructure Model*: This is the model of the physical system that includes wells, pipeline network and processing facilities.

(2) *The Contract Model*: This is the model of the production-sharing contracts and customer requirements.

Both submodels are network models with additional constraints and hence the overall model can also be viewed as two networks whose sources and sinks are coupled.

The following conventions are used in the model description:

(1) The constraints are numbered consecutively throughout the work (including supplementary material). Any non-numbered expression is not a constraint in the model and is an intermediate equality/inequality or the value of a constant or the bounds on a variable.

(2) All Greek letters denote parameters in the model with the exception of the universal gas constant R.

(3) Lower and upper case Roman letters denote decision variables.

(4) Superscripts U and L imply upper and lower bounds respectively.

A description of symbols that appear often in the model is presented in Table 1.

The Infrastructure Model

The infrastructure model describes the physical system. In particular, from a modeling perspective, it can be broken down into:

(1) *The trunkline network model*: this is the model of the flow network that includes pipelines and subsea connections.

(2) *The well performance model*: this represents pressure-flowrate relationships in the wells.

(3) *The compression model*: the compression model is the calculation of the power consumed by compressors.

Assumptions

Following are the primary assumptions in the infrastructure model:

(1) The gas is assumed to be ideal at standard conditions. Standard conditions (in the natural gas industry) are defined as the conditions at which volumetric flowrate is metered. The standard conditions in this work are taken as 15°C and 1 atm, which is close to the industry standard.

(2) The reservoir pressure is assumed to be constant over the planning period. This is justified by the planning period length of a few days to a few weeks over which the reservoir pressure does not change substantially.

(3) The composition of the reservoir fluid for a field is assumed invariant over the planning period. This is justified by the same argument as above. This assumption implies that the composition of gas from fields and the condensate gas ratio (CGR) stays the same over the planning period.

(4) Perfect mixing is assumed at junctions, since the network is currently operated without any preferential routing. The same assumption is also employed in the current production planning methodology being used for the SGPS.

Trunkline network model

The trunkline network is modeled as a *directed graph*. The nodes in this graph are fields, well platforms, riser platforms and LNG plants. The trunklines and subsea connections (for platforms serving several fields) are modeled as arcs of this graph. Let $(\mathcal{N}, \mathcal{A})$ be the directed graph representation of the trunkline network where $\mathcal{A}$ is the set containing all arcs and $\mathcal{N}$ is the set containing all nodes.

Flow Model. As described earlier, a reasonable prediction of pressure is essential for a planning model to be useful. The flow in pipelines is a mixture of gas and NGL. A full multiphase flow model cannot be employed in conjunction with current state-of-the-art global optimization algorithms that require an explicit functional representation of constraints. Hence, the standard gas flow equation[42] (described later) is used as a flow model. The pressure drop constant in this equation can be estimated from historical operating data. It has been observed that this relationship works well for long trunklines (>20 km) under steady-state operation. However, for short sections of pipelines or for pipelines where the flow fluctuates often, the predictions are not satisfactory and hence these are not modeled using this equation.

A representation of the network as in the model is presented in Figure 2. One should note the differences in topology of the networks represented in Figures 1 and 2. These differences facilitate easy representation of the pressure and flowrates constraints in a directed graph framework. The set of arcs $\mathcal{A}$ is partitioned into four subsets for the purposes of modeling the flow. Set $\mathcal{A}_q \subset \mathcal{A}$ denotes the set of arcs over which a volumetric flowrate variable $Q_{a,(i,j)}$ is defined.

(1) For most trunklines, the flow is described by the standard gas flow equation. This set is denoted by $\mathcal{A}_p \subset \mathcal{A}_q$. Therefore for this set

$$P_i^2 - P_j^2 = \kappa_{(i,j)}\, Q_{a,(i,j)}^2, \quad \forall (i,j) \in \mathcal{A}_p, \tag{1}$$

where P_i and P_j are pressures at the inlet and outlet, respectively, and $Q_{a,(i,j)}$ is the volumetric flowrate at standard conditions. This equation is one of the sources of nonconvexity in the model as it is a nonlinear equality. The coefficient $\kappa_{(i,j)}$ was estimated from the operating data for the SGPS, however it has been changed in the case study presented later as it is business sensitive information.

(2) The second set $\mathcal{A}_y \subset \mathcal{A}_q$ involves pipelines that can be shut off during normal operation. These pipelines are only a few kilometers in length and have a complicated configuration (e.g., multiple valves). Hence predictions from the standard gas flow equation do not match well with the operating data and therefore it is clearly not suitable to model these lines. These are modeled in the following way: when these lines are in the open state, a pressure inequality between the inlet and outlet is enforced and any flowrate up to the capacity of the lines is allowed. This is justified since the pressure drop across these lines is quite small (less than 0.1 MPa) at typical flowrates in the network. When the lines are in the closed state, the pressure inequality need not be enforced and the flowrate is pinned to zero.

To represent the above mathematically, a single binary variable $y^l_{(i,j)}$ per line is introduced such that $y^l_{(i,j)} = 1$, if the line is open and 0 otherwise. The resulting constraints can be reformulated as per Glover[43] that requires the introduction of two extra variables $w_{u,(i,j)}$ and $w_{d,(i,j)}$ to represent the upstream and downstream pressure respectively.

The following four constraints force $w_{u,(i,j)}$ to the upstream pressure P_i if $y^l_{(i,j)} = 1$ and to 0 if $y^l_{(i,j)} = 0$.

$$\begin{aligned} P_i - (1 - y^l_{(i,j)})\, P_i^U - w_{u,(i,j)} &\le 0, \quad \forall (i,j) \in \mathcal{A}_y, \\ w_{u,(i,j)} - P_i + (1 - y^l_{(i,j)})\, P_i^L &\le 0, \quad \forall (i,j) \in \mathcal{A}_y, \\ y^l_{(i,j)}\, P_i^L - w_{u,(i,j)} &\le 0, \quad \forall (i,j) \in \mathcal{A}_y, \\ w_{u,(i,j)} - y^l_{(i,j)} P_i^U &\le 0, \quad \forall (i,j) \in \mathcal{A}_y. \end{aligned} \tag{2}$$

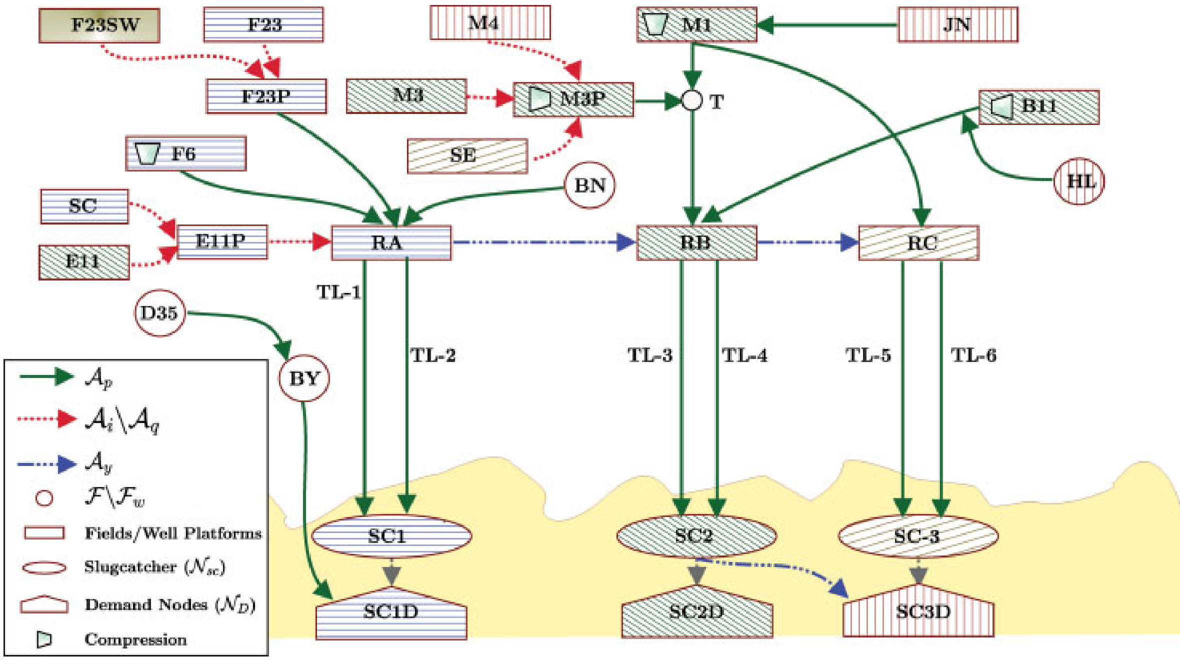

Figure 2. The SGPS trunkline network model: directed graph representation.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

Similarly the following four constraints force $w_{\mathrm{d},(i,j)}$ to the downstream pressure P_j if $y^l_{(i,j)} = 1$ and to 0 if $y^l_{(i,j)} = 0$.

$$\begin{aligned} P_j - (1 - y^l_{(i,j)})\, P^U_j - w_{\mathrm{d},(i,j)} \le 0, \quad \forall (i,j) \in \mathcal{A}_y, \\ w_{\mathrm{d},(i,j)} - P_j + (1 - y^l_{(i,j)})\, P^L_j \le 0, \quad \forall (i,j) \in \mathcal{A}_y, \\ y^l_{(i,j)}\, P^L_j - w_{\mathrm{d},(i,j)} \le 0, \quad \forall (i,j) \in \mathcal{A}_y, \\ w_{\mathrm{d},(i,j)} - y^l_{(i,j)} P^U_j \le 0, \quad \forall (i,j) \in \mathcal{A}_y. \end{aligned} \tag{3}$$

Finally, the following inequality represents the actual constraint relating the upstream and downstream pressure if $y^l_{(i,j)} = 1$. If $y^l_{(i,j)} = 0$, this constraint evaluates to 0 and hence is irrelevant

$$w_{\mathrm{d},(i,j)} - w_{\mathrm{u},(i,j)} \le 0, \quad \forall (i,j) \in \mathcal{A}_y. \tag{4}$$

Additionally, the following constraint forces the flowrate $Q_{a,(i,j)}$ to zero if $y^l_{(i,j)} = 0$ and otherwise keeps it within its bounds.

$$y^l_{(i,j)} Q^L_{a,(i,j)} \le Q_{a,(i,j)} \le y^l_{(i,j)} Q^U_{a,(i,j)}, \quad \forall (i,j) \in \mathcal{A}_y. \tag{5}$$

(3) There is a third set of arcs $\mathcal{A}_{\mathrm{sc}} \subset \mathcal{A}_q$ for which a constant pressure drop is assumed. This is because these arcs actually represent slugcatchers in LNG plants. The operational data suggest that the pressure drop across slugcatchers is constant. Then the pressure relationship between inlet and outlet pressures is simply

$$P_i - P_j = \Delta\pi_{(i,j)}, \quad \forall (i,j) \in \mathcal{A}_{\mathrm{sc}}, \tag{6}$$

where $\Delta\pi_{(i,j)}$ is the pressure drop associated with the facility.

(4) There is a set of trunklines which can be adequately represented by molar balances and pressure inequalities. These trunklines are subsea connections from fields to the well platforms (that serve multiple fields). This is justified since these lines have chokes that reduce the pressure to the common header level. Hence any pressure drop modeling for these lines is not important. These features imply that a flowrate variable need not be defined on these arcs since the sources at their origins (subsea fields) and ends (well platforms modeled as sources) can be directly related. Also there is no need to represent explicitly this subset since the constraints are already represented in the common molar balances and pressure inequalities.

Finally, a pressure inequality constraint is enforced over $\mathcal{A}_i \subset \mathcal{A}$, the set of all trunklines for which pressure at the inlet should always be greater than the pressure at the outlet. This is redundant for arcs in the set $\mathcal{A}_p \subset \mathcal{A}_i \cap \mathcal{A}_q$ (since it follows directly from the gas flow equation), but may be useful for strengthening relaxations.

$$P_j - P_i \le 0, \quad \forall (i,j) \in \mathcal{A}_i. \tag{7}$$

Material balances

The material balances are formulated in terms of molar flowrates of chemical species. This facilitates modeling of multiple qualities of gas (i.e., gas with different compositions) in the network. Eight species are modeled. The set of species is denoted by $\mathcal{S}$:

$$\mathcal{S} = \{\mathrm{CO_2}, \mathrm{N_2}, \mathrm{H_2S}, \mathrm{C_1}, \mathrm{C_2}, \mathrm{C_3}, \mathrm{C_4}, \mathrm{C_{5+}}\}.$$

The balances need to be formulated separately for junctions and for nodes that are sources or sinks. Let $\mathcal{N}_J$ be the set of nodes that are junctions. The set of nodes that are sources, i.e., that have volumetric production rate $Q_{s,i}$ and componentwise molar production rate $F_{s,i,k}$ associated with them is denoted as $\mathcal{N}_s$. However not all nodes in this set form the origin or destination of arcs in the set $\mathcal{A}_q$, the reason being that production from some of the sources is transferred to other sources directly and therefore the former nodes need not be on the sub-network defined by $\mathcal{A}_q$. The sources that are directly connected to this sub-network are denoted as set $\mathcal{N}_s$. With these definitions, the species molar balances at nodes can be easily formulated as:

$$F_{s,i,k} + \sum_{v:(v,i)\in\mathcal{A}} F_{a,(v,i),k} - \sum_{v:(i,v)\in\mathcal{A}} F_{a,(i,v),k} = 0, \quad \forall (i,k) \in \mathcal{N}_q \times \mathcal{S}, \tag{8}$$

$$\sum_{v:(v,i)\in\mathcal{A}} F_{a,(v,i),k} - \sum_{v:(i,v)\in\mathcal{A}} F_{a,(i,v),k} = 0, \quad \forall (i,k) \in \mathcal{N}_J \times \mathcal{S}, \tag{9}$$

where $F_{a,(i,v),k}$ denotes molar flow rate of species k on arc $(i,v) \in \mathcal{A}_q$ and $F_{s,i,k}$ denote the componentwise production rate from source $i \in \mathcal{N}_q$.

Let $\mathcal{F} \subset \mathcal{N}_s$ be the set of all fields and $\mathcal{N}_{wp} \subset \mathcal{N}_s$ be the set of well platforms. A node corresponding to a well platform that serves only one field and the node corresponding to that field are the same node and lie in $\mathcal{N}_{wp} \cap \mathcal{F}$. Denote the set of well platforms that serve multiple fields as $\mathcal{N}_{\mathrm{wp},m}$. Then production at well platforms in the set $\mathcal{N}_{wp,m}$ is given as

$$\sum_{j\in\mathcal{F}_i} F_{s,j,k} - F_{s,i,k} = 0, \quad \forall (i,k) \in \mathcal{N}_{wp,m} \times \mathcal{S}, \tag{10}$$

where set $\mathcal{F}_i$ is the set of fields connected to the platform $i \in \mathcal{N}_{wp,m}$.

Relationship with volumetric flowrate. The relationship between molar flowrate in arcs and volumetric flowrate (at standard conditions) is formulated using the ideal gas assumption. The total molar flowrate in an arc is proportional to the volumetric flowrate:

$$\sum_{k\in\mathcal{S}} F_{a,(i,j),k} - \phi\, Q_{a,(i,j)} = 0, \quad \forall (i,j) \in \mathcal{A}_q, \tag{11}$$

where ϕ is given by ideal gas equation of state:

$$\phi = 10^6 \frac{\pi_{\mathrm{sc}}}{R\theta_{\mathrm{sc}}},$$

where π_{sc} and θ_{sc} are the pressure and temperature, respectively, at standard conditions. At fields, a relationship between the molar production rate and volumetric flow rate is written:

$$F_{s,i,k} - \chi_{i,k}\,\phi\, Q_{s,i} = 0, \quad \forall (i,k) \in \mathcal{F} \times \mathcal{S}, \tag{12}$$

where $\chi_{i,k}$ is the mole fraction of species k in gas from field i. For well platforms that serve multiple fields, i.e., set $\mathcal{N}_{wp,m}$, and for demand nodes, i.e., set $\mathcal{N}_D$, the total molar rate should match the volumetric rates:

$$\begin{aligned} \sum_{k\in\mathcal{S}} F_{s,i,k} - \phi\, Q_{s,i} &= 0, \qquad \forall i \in \mathcal{N}_{wp,m}, \\ \sum_{k\in\mathcal{S}} F_{s,i,k} - \phi\, Q_{s,i} &= 0, \qquad \forall i \in \mathcal{N}_D. \end{aligned} \tag{13}$$

Splitters and mixers. There are both mixers and splitters of unknown composition in the system and hence bilinearities associated with models of splitting or mixing cannot be avoided. Nodes that are mixers do not require any special treatment since they are already modeled by molar balances. However, splitters do require special constraints to model them.

Let $\mathcal{N}_x$ be the set of splitters. Define

$$\mathcal{N}_{x,J} = \mathcal{N}_x \cap \mathcal{N}_J$$
$$\mathcal{N}_{x,q} = \mathcal{N}_x \cap \mathcal{N}_q.$$

Define a subset of arcs that are immediately downstream of splitters, i.e., $\forall (i,j) \in \mathcal{A}_q$ such that $i \in \mathcal{N}_x$. $s_{(i,j)}$ is the split fraction defined over a subset $\mathcal{A}_x$ of this set. $\mathcal{A}_x$ is defined by excluding exactly one arc corresponding to each splitter from the set defined earlier. $s_{(i,j)}$ varies between zero and one and represents the fraction that goes into arc (i,j) of the total flow coming into the splitter. It is not defined over one of the arcs downstream of a particular splitter since flow in that arc is implied by the molar balance constraints (Eqs. 8 and 9). At splitters that are junctions:

$$F_{a,(i,j),k} - s_{(i,j)} \sum_{v:(v,i)\in\mathcal{A}} F_{a,(v,i),k} = 0, \quad \forall (i,j,k) \in \{\mathcal{A}_x : i \in \mathcal{N}_{x,J}\} \times \mathcal{S}. \tag{14}$$

For splitters with a source term,

$$F_{a,(i,j),k} - s_{(i,j)} \left(\sum_{v:(v,i)\in\mathcal{A}} F_{a,(v,i),k} + F_{s,i,k} \right) = 0, \quad \forall (i,j,k) \in \{\mathcal{A}_x : i \in \mathcal{N}_{x,q}\} \times \mathcal{S}. \tag{15}$$

The above equalities are valid only with a perfect mixing assumption. However, this framework can be easily extended to represent preferential routing and blending in the network by defining mixing fractions for a particular outgoing arc over incoming arcs (instead of splitting fraction on outgoing arc), i.e., each outgoing arc can choose a fraction of different qualities of incoming gas. However for the SGPS, we choose not to model this scenario as it is not the case with the system.

The constraints arising from the models of splitters and mixers are the same as the classical pooling problem.[44,45] Therefore, some of the customized solution strategies that have been developed for the pooling problem may also be applied to this problem.

Well performance model

Let $\mathcal{W}$ denote the set of all wells. The well performance model comprises:

(1) The well flow model.
(2) The well material balances.

Well Flow Model. Variation of the reservoir pressure $\pi_{r,w}$ occurs in the vicinity of wells even if they belong to the same field. This variation corresponds to a pressure distribution in the reservoir that can be assumed invariant over a period of a few days to a few weeks. There are two pressures associated with each producing well. The *bottom hole pressure* $P_{b,w}$ is the pressure at the bottom of the well bore. The *flowing tubing head pressure* $P_{t,w}$ is the pressure at the well head. Following are the two relations that relate the well dry gas production rate $Q_{w,w}$ to the pressures $P_{b,w}$, $P_{t,w}$ and $\pi_{r,w}$:

(1) *In-flow performance* (IFP): This models the flow from the reservoir bulk to the well bore[42]:

$$\alpha_w Q_{w,w} + \beta_w Q_{w,w}^2 = \pi_{r,w}^2 - P_{b,w}^2, \quad \forall w \in \mathcal{W}. \tag{16}$$

Here α_w is Darcy's constant and β_w is the non-Darcy correction factor for modeling gas flow.

(2) *Vertical lift performance* (VLP): This models flow in the well bore itself:

$$\vartheta_w Q_{w,w}^2 = P_{b,w}^2 - \lambda_w P_{t,w}^2, \quad \forall w \in \mathcal{W}. \tag{17}$$

Additionally constraints can be enforced on the pressures from physical considerations. The bottom hole pressure should be less than the reservoir pressure for all wells:

$$P_{b,w} - \pi_{r,w} \leq 0, \quad \forall w \in \mathcal{W}. \tag{18}$$

The tubing head pressure must be less than the bottom hole pressure for all wells:

$$P_{t,w} - P_{b,w} \leq 0, \quad \forall w \in \mathcal{W}. \tag{19}$$

Implicit choke assumption

This assumption on the well head implies that the pressure at the common header (into which all wells produce) must be less than the flowing tubing head pressure for all wells connected to that header. In reality, this is achieved by a choke valve at each wellhead, however an explicit model of the choke valve is not considered here. The implicit choke provides the required pressure drop. This constraint needs to be formulated differently for platforms that have compression and those that produce directly into the trunkline network.

Let set $\mathcal{N}_{wp,c} \subset \mathcal{N}_{wp}$ denote the platforms that have compression. Note that $\mathcal{N}_{wp,c} \subset \mathcal{F}_w \cup \mathcal{N}_{wp,m}$, i.e., the nodes that represent platforms with compression are either well platforms serving a single field and therefore are the same as the node representing that particular field (as indicated earlier) or they are well platforms serving multiple fields which require compression. For these platforms, the compression inlet pressure $P_{c,i}$ is the common header pressure and this pressure must be less than or equal to the well head pressures of the wells producing to that platform:

$$P_{c,i} - P_{t,w} \leq 0, \quad \forall w \in \mathcal{W}_i, \quad i \in \mathcal{N}_{wp,c}. \tag{20}$$

For the set of fields for which well performance is modeled and that have no compression, i.e., the set $\mathcal{F}_{w,nc} \subset \mathcal{F}_w$, the common header pressure is the same as the pressure P_i of the node corresponding to the field,

$$P_i - P_{t,w} \leq 0, \quad \forall w \in \mathcal{W}_i, \quad i \in \mathcal{F}_{w,nc}. \tag{21}$$

Well Material Balances. Wells produce a mixture of gas, NGL (also termed condensates) and water. The NGL volume produced from a well is directly proportional to the volume of dry gas produced from that well and the CGR σ_w is assumed constant. This can be justified partially by the assumption on the constant composition of the reservoir fluid. Then the NGL production rate $Q_{Lw,w}$ from a well w is given as

$$Q_{Lw,w} = \sigma_w Q_{w,w}, \quad \forall w \in \mathcal{W}. \tag{22}$$

Finally total dry gas production from a field (for which well performance is modeled, i.e., it is in set $\mathcal{F}_w$), is the sum of productions from all wells that belong to that field (set $\mathcal{W}_i$):

$$Q_{s,i} = \sum_{w \in \mathcal{W}_i} Q_{w,w}, \quad \forall i \in \mathcal{F}_w. \tag{23}$$

The same is true for the total NGL production:

$$Q_{Ls,i} = \sum_{w \in \mathcal{W}_i} Q_{Lw,w}, \quad \forall i \in \mathcal{F}_w. \tag{24}$$

However, the transport of NGL through trunklines is not modeled due to impracticality of modeling multiphase flow through trunklines in a global optimization framework as discussed earlier. It is assumed that NGL produced can be transported to the demand nodes and that the transport of NGL does not limit the transfer of dry gas.

Compression model

It is assumed that the compression equation is given by the polytropic work of compression. The outlet pressure corresponds to the pressure of the node. Then the power of compression in MW is given by

$$W_i = \omega_i Q_{s,i} \left[\left(\frac{P_i}{P_{c,i}} \right)^v - 1 \right], \quad \forall i \in \mathcal{N}_{wp,c}, \tag{25}$$

where the constant ω_i is given by

$$\omega_i = \frac{1}{\eta_i} \frac{\zeta}{\zeta - 1} \frac{\pi_{sc}}{R\theta_{sc}} R\theta_{m,i} \frac{1}{\tau_{sec}} = \frac{1}{\eta_i} \frac{\pi_{sc}}{\theta_{sc}} \theta_{m,i} \frac{1}{v\tau_{sec}}, \quad \forall i \in \mathcal{N}_{wp,c},$$

and v is given as

$$v = \frac{\zeta - 1}{\zeta}.$$

Here π_{sc} and θ_{sc} are the pressure and temperature respectively at standard conditions, R is the universal gas constant, η_i is the compression efficiency, $\theta_{m,i}$ is the mean operating temperature of compression, τ_{sec} is number of seconds in a day and ζ is the polytropic constant for the process.

The power is constrained by the maximum rated power of a compressor as follows

$$\Psi_{L,i} \leq W_i \leq \Psi_{U,i}, \quad \forall i \in \mathcal{N}_{wp,c}.$$

Note that this is not treated as a constraint but instead as a bound. When the value of W_i at the optimal solution or at an intermediate point in the solution procedure is zero, the compression constraint 25 will be an equilibrium constraint. To avoid this, the lower bound $\Psi_{L,i}$ is set to a strictly positive small nonzero value. Physically, this means that compression stations should never be shut down and therefore the lowest production flowrates from these fields cannot go to zero, but only to a small value that ensures that the compressors are operating at their minimum power.

Table 2. Customer Requirements on Gas Quality

Specification	Constraint	$i \in \mathcal{N}_D$		
		SC1D	SC2D	SC3D
Γ_i^s(Excl. CO_2)	Range	53.0–56.0 (MJ/kg)	53.0–56.0 (MJ/kg)	53.0–56.0 (MJ/kg)
χ_{i,CO_2}^S	Less than	5.8 (mol %)	5.8 (mol %)	6.0 (mol %)
χ_{i,N_2}^S	Less than	1.2 (mol %)	1.0 (mol %)	1.2 (mol %)
χ_{i,H_2S}^S	Less than	250 (ppmV)	270 (ppmV)	30 (mg/m^3)
$\chi_{i,S}^S$	Less than	29.0 (mg/m^3)	22.0 (mg/m^3)	25.0 (mg/m^3)
χ_{i,C_2}^S (Excl. CO_2)	Greater than	3.2 (mol %)	3.4 (mol %)	4.0 (mol %)
χ_{i,C_3}^S (Excl. CO_2)	Greater than	2.1 (mol %)	2.7 (mol %)	3.0 (mol %)
χ_{i,C_4}^S (Excl. CO_2)	Less than	2.4 (mol %)	2.7 (mol %)	2.7 (mol %)
$\chi_{i,C_{5+}}^S$ (Excl. CO_2)	Less than	1.0 (mol %)	1.7 (mol %)	1.7 (mol %)

The Contract Modeling Framework

Contractual rules are central to the operation of the system as outlined earlier. Therefore, any supply chain planning tool needs to take these rules into account to be implementable on the real system. A framework for incorporating these rules is presented here. It is useful to view the rules as comprising the following four subcategories:

(1) *Demand rates and delivery pressures*: these are maximum and minimum supply rates and delivery pressures to the LNG plants.

(2) *Gas quality specifications*: these are customer requirements, e.g., the heating value and the composition of the feed gas to the LNG plants.

(3) *Field plant contract assignments*: these are production-sharing contracts that designate certain fields to supply a specific plant.

(4) *Operational rules*: operational rules enforce conditions on the network to ensure proper operation and implementation of PSC.

The operational rules are included here since the framework for modeling them is the same as the contractual rules and, moreover, several contractual rules also invoke operational constraints in order to implement them on the network.

Demand rates and delivery pressure

Contracts enforce a maximum demand rate at each LNG plant that reflects the maximum intake of the LNG plant and the supply cannot exceed this rate. Additionally, the demands are bounded from below by a minimum rate. Together, this defines a narrow window of operation as defined by the LNG plant operators. Since the demand nodes are sinks with a negative production rate, the bounds are reversed as follows

$$-\Lambda_{d,i}^U \leq Q_{s,i} \leq -\Lambda_{d,i}^L, \quad \forall i \in \mathcal{N}_D,$$

where $\Lambda_{d,i}^U$ and $\Lambda_{d,i}^L$ are maximum and minimum demand rates respectively. These requirements are enforced as bounds and not as constraints.

The primary means of control for the network is to regulate the pressure at the slugcatchers. Moreover, the LNG plants require the feed gas to be at a certain pressure for proper operation. If the pressure is too low, the gas flowrate has to be cut down to maintain the pressure. If it is too high, the liquids fail to separate out completely from two phase mixture in the slugcatchers and a lot of liquid goes through to the LNG plants, which is undesirable since in normal circumstances NGL belongs to the upstream operator, but in this case, LNG plants will take a share and hence a loss for the upstream operator:

$$\pi_{d,i}^L \leq P_i \leq \pi_{d,i}^U, \quad \forall i \in \mathcal{N}_{\mathrm{sc}},$$

where $\pi_{d,i}^L$ and $\pi_{d,i}^U$ are lower and upper limits of the operating pressure at slugcatchers. This constraint is also represented as a bound in the model.

Gas quality specifications

These specifications are given by the sales agreement between the upstream operators and the LNG plant operators and comprise constraints on species concentrations and gross heating value. An example of specifications as outlined in the contracts is presented in Table 2 for the demand nodes in the network in Figure 2.

Gross Heating Value. The most important gas quality specification is the gross heating value (GHV) of the feed gas to the LNG plants, since it has a direct effect on the calorific value of the LNG produced. This has implications for the reputation of the LNG suppliers. The gross heating value of the feed gas is measured in terms of the heat of combustion per unit mass of gas (excluding CO_2 as it is separated before liquefaction of the gas). The GHV is specified in a range.

For this work, the GHV is constrained to be higher than the lower limit of the range. Energy content of the gas is calculated using the superior calorific values of C_1 through C_{5+} at a prespecified temperature and pressure. In this work, we assumed it to be calculated at 15°C and 1 atm. The superior calorific values at 15°C and 1 atm were obtained from the ISO standard for calculation of calorific values of natural gas.[46] For C_1 through C_3, the superior calorific values for methane, ethane and propane respectively are used. For C_4, the mean of superior calorific value of *n*-butane and *2*-methylpropane is used. For heavier components, the heating value and molecular weight of *n*-hexane is used. The inequality representing the GHV constraint is given as:

$$\sum_{k \in \mathcal{S}_h} \gamma_k \mu_k F_{s,i,k} - \Gamma_i^s \sum_{k \in \mathcal{S} \setminus CO_2} \mu_i F_{s,i,k} \leq 0, \quad \forall i \in \mathcal{N}_D, \qquad (26)$$

where γ_k is the superior calorific value of component k on a per unit mass basis, μ_k is the molecular weight of the component, Γ_i^s is the lower limit of GHV specification at demand i and $\mathcal{S}_h$ is the set of species that are used to calculate heating value. It should be noted that the inequality is the opposite of what may seem intuitive since the demand nodes are sinks and all molar rates are negative. The heating values and molecular weights used for the GHV calculation are presented in the supplementary material.

Composition Specifications. Additional quality specifications are related to maintaining the capacity and ensuring proper operation of the LNG plants. For example, if the CO_2 fraction in the feed gas goes above a certain threshold, the plants have to cut back the total amount of feed gas being processed because the capacity of CO_2 extraction units is exceeded.

It should be noted that the inequalities are the opposite of what may seem intuitive since the molar rates are negative at demand nodes:

(1) The CO_2 and N_2 mole fractions should be less than the threshold:

$$10^{-2}\chi^s_{i,k}\sum_{j\in\mathcal{S}}F_{s,i,j} - F_{s,i,k} \leq 0, \quad \forall(i,k)\in\mathcal{N}_D\times\{\mathrm{CO_2},\mathrm{N_2}\}. \tag{27}$$

(2) The amount of sulfur (mass per unit volume) should be below specified limits:

$$10^{-6}\chi^s_{i,\mathrm{S}}Q_{s,i} - \mu_{\mathrm{S}}F_{s,i,\mathrm{H_2S}} \leq 0, \quad \forall i\in\mathcal{N}_D. \tag{28}$$

(3) As is evident from Table 2, different demand nodes have different units in which the hydrogen sulfide concentration specification is expressed. The hydrogen sulfide (H_2S) concentration should be less the specified ppmV (parts per million by volume) for the set $\mathcal{N}_{D,HSP}\subset\mathcal{N}_D$:

$$10^{-6}\chi^s_{i,\mathrm{H_2S}}\phi Q_{s,i} - F_{s,i,\mathrm{H_2S}} \leq 0, \quad \forall i\in\mathcal{N}_{D,HSP}. \tag{29}$$

The hydrogen sulfide (H_2S) concentration should be less the specified mg/m^3 for the set $\mathcal{N}_{D,HSM}\subset\mathcal{N}_D$:

$$10^{-6}\chi^s_{i,\mathrm{H_2S}}Q_{s,i} - \mu_{\mathrm{H_2S}}F_{s,i,\mathrm{H_2S}} \leq 0, \quad \forall i\in\mathcal{N}_{D,HSM}. \tag{30}$$

(4) The C_2, C_3 mole fractions excluding CO_2 should be greater than the threshold:

$$F_{s,i,k} - 10^{-2}\chi^s_{i,k}\sum_{j\in\mathcal{S}\backslash\mathrm{CO_2}}F_{s,i,j} \leq 0, \quad \forall(i,k)\in\mathcal{N}_D\times\{\mathrm{C_2},\mathrm{C_3}\}\cdot \tag{31}$$

(5) The C_4, C_{5+} mole fraction excluding CO_2 should be less than the threshold:

$$10^{-2}\chi^s_{i,k}\sum_{j\in\mathcal{S}\backslash\mathrm{CO_2}}F_{s,i,j} - F_{s,i,k} \leq 0, \quad \forall(i,k)\in\mathcal{N}_D\times\{\mathrm{C_4},\mathrm{C_{5+}}\}\cdot \tag{32}$$

Production-sharing contracts

As discussed earlier, several parties have stakes in different parts of the system and therefore a complicated framework of PSC (referred to as simply contracts later) exists. A plant cannot receive supply from arbitrary fields. It can only receive supply from fields that are "produced" under the "contract" authorized to supply it. A framework for modeling these contracts is presented here.

Terminology. Every field in the system is associated with a contract. In the industry terminology, "the field is produced under a contract." A contract can contain several fields. The sum of productions from the fields associated with a particular contract is the supply of that contract. A contract is mandated to *supply* to a particular demand (a LNG plant). This is termed the *primary demand* of the contract. *Inter-contract transfers* are exchanges of gas volumes between different contracts. They are required because the primary demand and supply of a contract may not match either because of production infrastructure constraints or customer quality specifications. A contract is in *excess* if its supply exceeds its primary demand and it is in *deficit* otherwise. This is the state of the contract (contract state). The *inter-contract transfer rules* are the set of rules that govern the inter-contract transfers. Additionally, these rules may also invoke operational rules to implement the transfers on the pipeline network. When a transfer takes place as per a rule, the rule is said to have been *activated* (also termed the state of an inter-contract transfer (rule) is *active*).

Challenges in Representation. The primary challenges in representing these contracts in a mathematical programming framework are as follows.

The inter-contract transfer rules are activated by a particular contract being in deficit or excess and, in certain cases, based also on several additional conditions that represent priorities and operational concerns. Hence, they are based on logical conditions and therefore require binary variables and constraints to model them.

The rules also interact with each other. This is because to establish whether supply of gas is available from a contract, it is not sufficient to know the primary demand and supply of the contract, but also whether other rules have activated and borrowed from the contract under consideration.

Modeling the inference of a rule

The way a rule is stated in the operational documents for the real system may not cover all logical possibilities. This is because to a human operator, the logic and inference of a rule are obvious. Hence, the statement of a rule may not dictate the outcomes that can be easily deduced by a human.

An example of this difficulty is as follows. The deficit rules state that when a contract is in deficit, what other contracts must supply it. However, they do not explicitly prohibit transfers to contracts in excess since this is obvious to a human operator. Mathematically, deficit rules as stated only provide sufficient conditions for transfer activations. However, they may or may not be necessary for transfer activation and this missing information is not stated explicitly since a human operator can deduce it. Hence an exact mathematical representation of deficit rules will only model the sufficiency clause. This will lead to a problem when the clause was indeed necessary, generating feasible operations where the clause is false, but the transfer activates. This operation would be deemed infeasible by a human operator.

Though the above example may seem obvious, for certain rules and scenarios, it may not be clear if an exact representation of the inference of a rule has been embedded in the model or if more logical constraints should be added to avoid infeasible scenarios at the solution. A rigorous verification may require evaluating all feasible integer realizations of the

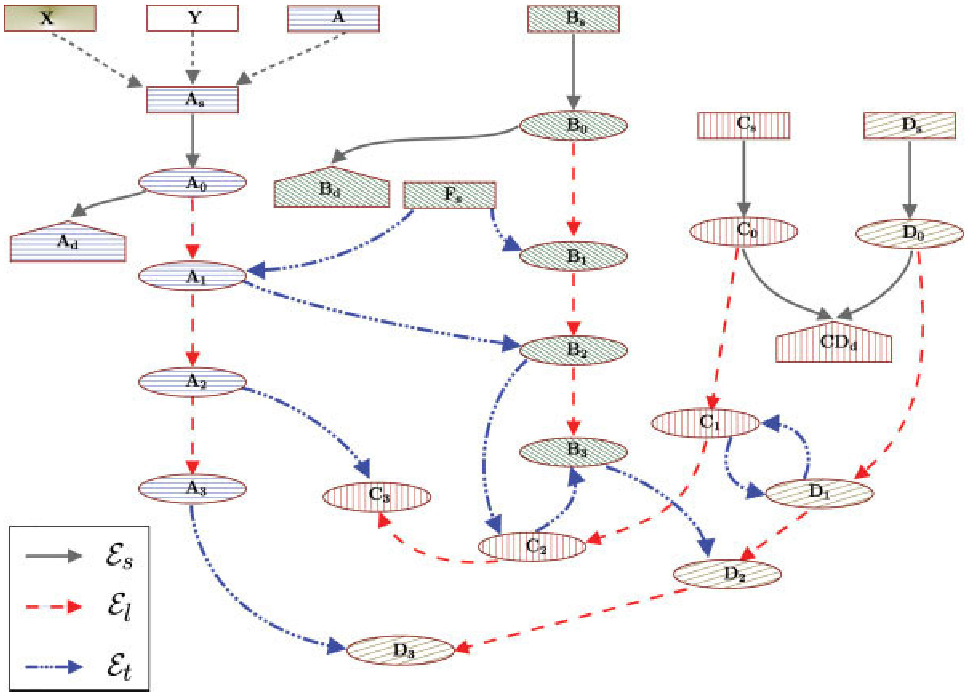

Figure 3. Contracts in the case study: network representation.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

problem which is clearly impractical. The modeling approach in this work has been to add as many logical constraints as were obvious or had become apparent during the development of the model at that stage. The solution was then evaluated for any infeasibility and if it was found that solution would be deemed infeasible by a human, additional logical constraints were added to avoid such a scenario and iterate until the solution was satisfactory.

Contracts: Network Representation. Contractual rules interact with each other and therefore there are levels of excesses/deficits in a contract. *Excess (or deficit) from a particular contract at the nth level* is defined as the excess (or deficit) after *n* excess (or deficit) rules for that particular contract have been considered. It is not necessary that the rules must have been activated.

This formulation can be represented using a directed graph $(\mathcal{L}_l, \mathcal{E}_l)$ with nodes of the graph (set $\mathcal{L}_l$) indicating the levels in each contract and flows in arcs (set $\mathcal{E}_l$) indicating excesses or deficits between the levels. The flow in an arc is permitted to be either positive (flow direction same as the arc direction) or negative (flow direction opposite to the arc direction) as opposed to traditional network theory. A positive flow incident out of a particular node indicates excess in the corresponding level. On the other hand, a negative flow indicates deficit corresponding to that level and this therefore means that the flow is incident into the node (i.e., the flow direction is opposite to the arc originating at that node) corresponding to that particular level.

Each contract has a single source node corresponding to it in the contract graph whose production rate is equal to the sum of production rates of fields producing under that contract. Sink nodes in the infrastructure network have counterparts in the contract graph representing LNG plants. Together sources and sinks form the set $\mathcal{L}_s$. The supply and demand arcs connect the source nodes and the demand nodes respectively to the contract levels sub-graph $(\mathcal{L}_l, \mathcal{E}_l)$. Together the supply and demand arcs form the set $\mathcal{E}_s$.

All nodes that correspond to one particular contract can be grouped to indicate the levels of excess or deficit in this contract. The nodes are labeled with the index of the contract and level, so that node corresponding to contract p at level i is $p_i \, (\in \mathcal{L}_l)$. The supply nodes and demand nodes are termed p_s and $p_d \, (\in \mathcal{L}_s)$, respectively.

The inter-contract transfer rules can now be represented as arcs between the nodes from different contracts. For example, a transfer from contract p to q may be represented as a unique arc (p_i, q_j) on this graph where i and j are determined by the priority of that transfer over other transfers. The set of these arcs is represented by $\mathcal{E}_l$.

It should be noted that the originating node p_i for a transfer (p_i, q_j) from contract p to q can be a supply node (i.e., $i = s$) under special conditions. This is because there are fields in the system that can supply to multiple PSC and hence need to be treated as special cases. These are modeled by treating them as a separate contract that has no explicit mandated demand. In such a case, there is no need for supply or demand arcs and the states corresponding to these fields supplying one or the other contract are represented by transfer arcs originating at the corresponding supply node.

The contract network representation is given by the directed graph $(\mathcal{L}, \mathcal{E}) = (\mathcal{L}_s \cup \mathcal{L}_l, \mathcal{E}_s \cup \mathcal{E}_l \cup \mathcal{E}_t)$. The excess and deficit calculations at different levels (including inter-contract transfers) are equivalent to material balances on this network. Figure 3 represents an example of the contract network representation which is similar in complexity to one on the real system, though not identical for confidentiality reasons.

It contains four contracts A, B, C and D and three demands. F corresponds to a field that can supply multiple contracts. This network is used in the case study presented later.

Excess/Deficit Policy. The actual constraints for activating the excess/deficit rules can be formulated within the contract network framework presented in the previous section. An atomic proposition representing the contract state is formulated which is true when the contract is in excess and false otherwise.

Once a contract is in excess (or deficit) at a particular level, it should maintain that state for all further levels following it. If this is not so, there is a possibility of transfers that should not be normally permitted. For example, consider the following two rules that state:

- If A is in excess and B is in deficit, then A should supply B.
- If B is in excess and C is in deficit, then B should supply C.

It is possible to get a scenario, when A is in excess and both B and C are in deficit. A can supply B to make it in excess and B now can supply C. Hence A indirectly supplies C, although this was never intended by the two rules as stated. The policy stated ensures that this cannot happen. This example also demonstrates the difficulty outlined previously that a literal modeling of rules may not be sufficient to represent the inference of the rules.

The above policy implies that a single atomic proposition is needed for representing the contract state for most contracts, i.e., the flow from only a single arc corresponding to a contract in the contract network representation needs to be examined to ascertain if that contract is in excess (or deficit). Notationally, atomic proposition $E_{(p_i,p_{i+1})}$ is true if the flow in arc (p_i, p_{i+1}) is nonnegative in the contract network representation and this is equivalent to contract p being in excess. The level i used for defining contract state is 0 for most contracts, i.e, the excess flag is set before any transfer rules have activated (therefore, the excess flag for a contract p is given by $E_{(p_0,p_1)}$ for most cases).

However, for some contracts, there is a need to define multiple atomic propositions to represent excess. This is because these contracts contain fields that can supply to other contracts under special conditions. These fields feed downstream of the node corresponding to 0th level in the contract network and hence flow in arc (p_0,p_1) is not sufficient to resolve the state. This arrangement exists because a field in one contract system may be physically connected to the other contract system and hence might require special rules. In such a case, there are several atomic propositions $E_{(p_i,p_{i+1})}$ each corresponding to a different arc $(p_i, p_{i+1}) \in \mathcal{E}_{l,a}$.

Transfer State. Atomic proposition $T_{p,q}$ is used to indicate the state of an inter-contract transfer of gas from contract p to contract q. $T_{p,q}$ is true implies that the transfer takes place (or the transfer is activated). It is equivalent to the flowrate on an inter-contract transfer arc (p_i,q_j) being non-negative, (p_i, q_j) being the unique arc in the contract network representing transfer flowrate from contracts p to q.

Transfer Priorities. When a contract p is in excess and there are several contracts (say q, r) that are in deficit and should receive supply from contract p (as per the rules), the rules define priorities of transfer because the excess may not be enough to fulfill all the transfer demands being placed on contract p. If these transfer priorities are not modeled, a solution may violate these priorities (though the balances will still be closed and all deficits will be fulfilled but not as per the priorities dictated by the rules).

Atomic propositions $S_{(q_j,q_{j+1})}$ need to be defined corresponding to the transfer from contract p to contract q (arc (p_i, q_j) or atomic proposition $T_{p,q}$). $S_{(q_j,q_{j+1})}$ flags the state of the receiving contract once a transfer has been made. It being true implies that arc (q_j, q_{j+1}) has a non-negative flowrate and hence the transfer has fulfilled the deficit. Another transfer $T_{p,r}$ from the supply contract p can only go ahead if the proposition $S_{(q_j,q_{j+1})}$ is true.

However, it is important to note that a $S_{(r_j,r_{j+1})}$ need not be defined corresponding to every transfer $T_{p,r}$. Specifically, it is not required for transfers that are at the lowest priority on the supplying side, i.e., for transfer arc (p_i, r_j), p_i is the terminal level node (the last node representing level) corresponding to contract p in the contract network, because there is no transfer rule that follows it and needs to depend on it. Of course this lowest priority transfer rule still needs to honor all the previously defined priorities. Another condition when the priority flag is not required is when a receiving contract r has the transfer $T_{p,r}$ at the lowest priority, i.e., r tries to borrow from p, only when everything else fails. In this case r_j is a terminal level node and as per material balance on the network, the demand should be satisfied. To summarize, when either p_i or r_j is a terminal level node, it is not required to define a priority corresponding to the transfer arc connecting them. We define the set of arcs over which priority atomic propositions are defined as $\mathcal{E}_{l,S} \subset \mathcal{E}_l$.

Coupling Constraints between Infrastructure and Production-Sharing Contracts Models. Equations 33–35 represent the coupling constraints between the infrastructure and the contract networks. These are essentially constraints linking the sources and sinks of both networks. If multiple contracts have no special rules distinguishing them from one of the others in the group (i.e., they either have no specific rules or have the same rules as the others), they are collapsed to represent a single source and are not represented separately on the contract network. Let $\mathcal{C}^S$ be the superset of all contracts, some of which may not be represented on the contract network. The set of contracts that are actually represented is $\mathcal{C} \subset \mathcal{C}^S$. These sets for the example shown in Figure 3 are defined as follows:

$$\mathcal{C}^S = \{\mathrm{A, B, C, D, F, X, Y}\},$$
$$\mathcal{C} = \{\mathrm{A, B, C, D, F}\}.$$

Set $\mathcal{C}^S$ is maintained in the model for future flexibility so as to add additional rules, at which point the contract network can be expanded to incorporate those contracts. Denote the set of contracts in the set $\mathcal{C}^S$ that are collectively represented as a single source $p_s \in \mathcal{C}$ in the contract network as $\mathcal{C}^S_p$.

The supply rate of a contract $q_{c,i}$ is given by

$$q_{c,i} = \sum_{j \in \mathcal{F}_i} Q_{s,j}, \quad \forall i \in \mathcal{C}^S, \tag{33}$$

where $\mathcal{F}_i$ is the set of fields producing under contract i. The source rates for the contract network are given by

$$q_{s,p_S} = \sum_{j \in \mathcal{C}_p^S} q_{c,j}, \quad \forall p_S \in \mathcal{C}. \tag{34}$$

The sink nodes in the contract network are mapped to the demand nodes in the infrastructure network

$$q_{s,u_i} = Q_{s,i}, \quad \forall i \in \mathcal{N}_{\mathrm{D}} \backslash \{i\}, \tag{35}$$

where u_i is the demand node in the contract network corresponding to the demand node i in the infrastructure network. This equation should not be formulated for exactly one demand node in the infrastructure network. The reason is that the last equality is implied by the combined material balances in the infrastructure and contract networks. If this is included, it violates the linear independence constraint qualification and has been observed to create severe problems for convergence.

Volumetric Balances in the Contract Network Representation. Volumetric balances in the contract network represent the actual constraints for supply and demand balances as well as for the excess/deficit calculations for each contract. Volumetric balances are easier to formulate over the entire contract network than referring to individual contracts. Hence the convention of referring to a node as p_i and it being associated with contract p has been dropped in this section.

Let $e_{(u,v)}$ be the excess volumetric flowrate in arc $(u, v) \in \mathcal{E}_l$. It is positive when a contract is in excess and negative otherwise. Let $t_{(u,v)}$ represent the transfer rates in arcs $(u, v) \in \mathcal{E}_t$. Furthermore $q_{a,(u,v)}$ represents flowrates in arcs $(u, v) \in \mathcal{E}_s$, i.e., supply and demand arcs. At the nodes that are junctions in the contract network (i.e., all nodes except the supply and demand nodes, same as set $\mathcal{L}_l$), the balance can be represented as:

$$\sum_{v:(v,u)\in\mathcal{E}_l} e_{(v,u)} - \sum_{v:(u,v)\in\mathcal{E}_l} e_{(u,v)} + \sum_{v:(v,u)\in\mathcal{E}_t} t_{(v,u)} - \sum_{v:(u,v)\in\mathcal{E}_t} t_{(u,v)} + \sum_{v:(v,u)\in\mathcal{E}_S} q_{a,(v,u)} - \sum_{v:(u,v)\in\mathcal{E}_S} q_{a,(u,v)} = 0, \qquad \forall u \in \mathcal{L}_l. \tag{36}$$

Nodes that are supply or demand nodes to the network (i.e., that are not junctions) have a production term. It should be noted that these nodes do not have arcs representing levels terminating at or originating from them.

$$\sum_{v:(v,u)\in\mathcal{E}_S} q_{a,(v,u)} - \sum_{v:(u,v)\in\mathcal{E}_S} q_{a,(u,v)} + \sum_{v:(v,u)\in\mathcal{E}_t} t_{(v,u)} - \sum_{v:(u,v)\in\mathcal{E}_t} t_{(u,v)} + q_{s,u} = 0, \qquad \forall u \in \mathcal{L}_s. \tag{37}$$

Relationship Between Atomic Propositions and Flowrates in Contract Network. A binary variable is introduced in the model corresponding to each atomic proposition defined previously. The atomic proposition being true is equivalent to the binary variable being equal to 1.

Binary variable $y^e_{(p_i,p_{i+1})}$ corresponds to the atomic proposition $E_{(p_i,p_{i+1})}$ and is the contract state binary variable. It should be noted again that this variable is not defined over every arc representing excess or deficit in the system. The set of arcs over which this binary variable is defined is termed $\mathcal{E}_{l,a} \subset \mathcal{E}_l$.

Let $e_{(p_i,p_{i+1})}$ denote the excess rate for contract p at level i. $e_{(p_i,p_{i+1})}$ is negative if the supply of contract is less than the primary demand of contract p and the contract is in deficit. The binary variables $y^e_{(p_i,p_{i+1})}$ are related to the flowrates in the excess arcs $e_{(p_i,p_{i+1})}$ over the set of all contracts $\mathcal{C}$ as follows:

$$\begin{aligned} e^L_{(p_i,p_{i+1})} - y^e_{(p_i,p_{i+1})} e^L_{(p_i,p_{i+1})} - e_{(p_i,p_{i+1})} &\le 0, \quad \forall (p_i, p_{i+1}) \in \mathcal{E}_{l,a}, \\ e_{(p_i,p_{i+1})} - y^e_{(p_i,p_{i+1})} e^U_{(p_i,p_{i+1})} &\le 0, \quad \forall (p_i, p_{i+1}) \in \mathcal{E}_{l,a}, \end{aligned} \tag{38}$$

where $e^L_{(p_i,p_{i+1})}$ and $e^U_{(p_i,p_{i+1})}$ are lower and upper bounds respectively to $e_{(p_i,p_{i+1})}$. These constraints ensure that $y^e_{(p_i,p_{i+1})} = 1$ is equivalent to $0 \le e_{(p_i,p_{i+1})} \le e^U_{(p_i,p_{i+1})}$ and $y^e_{(p_i,p_{i+1})} = 0$ is equivalent to $e^L_{(p_i,p_{i+1})} \le e_{(p_i,p_{i+1})} \le 0$. These constraints therefore couple the contract state binary variables with the actual flowrates in the contract network representation.

$e^L_{(p_i,p_{i+1})}$ is strictly negative and is set by making the assumption that the supply of contract p is zero and the primary demand of contract p is fulfilled exclusively by inter-contract transfers and is therefore the maximum deficit rate that is possible for a contract. It is therefore set to the lower bound of the rate at the sink node corresponding to contract p (physically this is the maximum demand rate at the LNG plant corresponding to this sink). On the other hand, $e^U_{(p_i,p_{i+1})}$ is the maximum excess rate possible and is set assuming that the primary demand of the contract is zero and all the supply is available for inter-contract transfers. It is therefore equal to the upper bound of the supply of the contract and is strictly positive. A further discussion can be found in the supplementary material.

An exactly analogous constraint exists for priority atomic propositions. Let $y^S_{(q_j,q_{j+1})}$ correspond to atomic proposition $S_{(q_j,q_{j+1})}$. Then

$$\begin{aligned} e^L_{(q_j,q_{j+1})} - y^S_{(q_j,q_{j+1})} e^L_{(q_j,q_{j+1})} - e_{(q_j,q_{j+1})} &\le 0, \quad \forall (q_j, q_{j+1}) \in \mathcal{E}_{l,S}, \\ e_{(q_j,q_{j+1})} - y^S_{(q_j,q_{j+1})} e^U_{(q_j,q_{j+1})} &\le 0, \quad \forall (q_j, q_{j+1}) \in \mathcal{E}_{l,S}. \end{aligned} \tag{39}$$

This constraint performs exactly the same function as Eq. 38 and bounds on $e_{(q_j,q_{j+1})}$ are calculated as above and they satisfy the same properties.

Let $t_{(p_i,q_j)}$ denote the transfer rate between contract p and q (i.e., the flowrate in arc $(p_i, q_j) \in \mathcal{E}_t$). Also, let $y^t_{p,q}$ be the binary variable corresponding to the atomic proposition $T_{p,q}$ representing the state of the (p,q) transfer such that $y^t_{p,q} = 1$ when $T_{p,q}$ is true. Then this binary variable $y^t_{p,q}$ can be coupled with the actual transfer flowrate $t_{(p_i,q_j)}$ by the following relationships

$$\begin{aligned} - t_{(p_i,q_j)} &\le 0, \quad \forall (p_i, q_j) \in \mathcal{E}_t, \\ t_{(p_i,q_j)} - y^t_{p,q} t^U_{(p_i,q_j)} &\le 0, \quad \forall (p_i, q_j) \in \mathcal{E}_t. \end{aligned} \tag{40}$$

It should be noted that the upper bound for $t_{(p_i,q_j)}$ is set to $e^U_{(p_{i-1},p_i)}$ since the maximum amount of transfer that is possible is the maximum flowrate possible in the excess arc upstream of node p_i in the transferring contract (except for transfer arcs originating at supply nodes in which case it is set to the upper bound on the production rate from that supply node). Explicit representation of these bounds can be found in the supplementary material. These constraints ensure that $t_{(p_i,q_j)} = 0$, i.e., no transfer takes place when $y^t_{p,q} = 0$ and $t_{(p_i,q_j)} \geq 0$ when $y^t_{p,q} = 1$.

Transfer-Activation Constraints. An inter-contract transfer rule can now be expressed in terms of a logical expression involving atomic propositions representing contract states, the transfer states, the transfer priorities flags and operational flags. This logical expression can be converted to its conjunctive normal form (CNF) and this CNF can be converted to constraints involving binary variables.[47,48] Following is an example from the case study presented in this work. A brief description of contract rules used in the case study can be found in the Appendix and further details can be found in the supplementary material.

Example (Transfer-Activation Constraint). Consider the following contractual rule:

A to B Transfer Rule: *When demand at LNG 2 (SC2D) cannot be fulfilled by contract B, "borrow gas" from contract A shall supply to LNG 2 (SC2D) to meet the demand and the RA to RB trunkline shall be open at this stage.*

The following atomic propositions are required:

- $E_{(A_0,A_1)}$: A is in excess (contract A state),
- $E_{(B_1,B_2)}$: B is in excess (contract B state),
- $T_{A,B}$: A supplies B (transfer state),
- $C_{(RA,RB)}$: RA to RB trunkline is open (additional operational state).

The logic in the above rule can be expressed as:

$$(E_{(A_0,A_1)} \wedge \neg E_{(B_1,B_2)}) \Rightarrow (T_{A,B} \wedge C_{(RA,RB)}).$$

Here $\wedge, \vee, \neg$, and $\Rightarrow$ are logical AND, OR, NOT, and IMPLICATION operators respectively. The above is equivalent to

$$\neg(E_{(A_0,A_1)} \wedge \neg E_{(B_1,B_2)}) \vee (T_{A,B} \wedge C_{(RA,RB)}).$$

The CNF of the statement is:

$$(\neg E_{(A_0,A_1)} \vee E_{(B_1,B_2)} \vee T_{A,B}) \wedge (\neg E_{(A_0,A_1)} \vee E_{(B_1,B_2)} \vee C_{(RA,RB)}).$$

The CNF can be converted to binary constraints

$$1 - y^e_{(A_0,A_1)} + y^e_{(B_1,B_2)} + y^t_{A,B} \geq 1,$$
$$1 - y^e_{(A_0,A_1)} + y^e_{(B_1,B_2)} + y^l_{(RA,RB)} \geq 1,$$

so that the final form of the constraints representing the A to B transfer rule is as follows:

$$y^e_{(A_0,A_1)} - y^e_{(B_1,B_2)} - y^t_{A,B} \leq 0,$$
$$y^e_{(A_0A_1)} - y^e_{(B_1,B_2)} - y^l_{(RA,RB)} \leq 0.$$

Additional Logical Constraints. Additional logical constraints must be added to formulate a satisfactory representation of the contracts as well as to strengthen relaxations. These are listed as follows:

(1) Cuts should be added by analyzing the negation of transfer-activation conditions. If the negation corresponds to a condition when the transfer should not activate (i.e., the original transfer-activation condition was both necessary and sufficient for the transfer to take place), an additional logical constraint which states that no transfer should take place when the negation is true must be added to the problem. It is important to note that for some transfers, a negation of activation condition does not imply that the transfer cannot activate and hence the above logical cut will be invalid. The negation of transfer rules also introduces logical constraints that imply that no transfer should be made to a contract in excess and hence this need not be enforced explicitly.

(2) If a contract q requiring a single excess flag is in excess, i.e, the unique excess flag $E_{(q_i,q_{i+1})}$ corresponding to it is true, then any priority atomic propositions $S_{(q_j,q_{j+1})}$ defined for that contract are also true. Also, if a contract q has multiple excess flags, then a priority flag $S_{(q_j,q_{j+1})}$ is true if the closest excess flag to $S_{(q_j,q_{j+1})}$ is true (i.e., $E_{(q_{i_j},q_{i_j+1})}, (q_{i_j}, q_{i_j+1}) \in \mathcal{E}_{l,a}$, is true, (q_{i_j}, q_{i_j+1}) is the closest element of set $\mathcal{E}_{l,a}$ to node q_j in the sub-graph of contract q). This can be represented logically as follows

$$E_{(q_{i_j},q_{i_j+1})} \Rightarrow S_{(q_j,q_{j+1})}, \ \ i_j < j, \ \ \forall (q_{i_j},q_{i_j+1}) \in \mathcal{E}_{l,a}, \ \ (q_j,q_{j+1}) \in \mathcal{E}_{l,S},$$

where (q_{i_j}, q_{i_j+1}) is the closest arc in set $\mathcal{E}_{l,a}$ to node q_j. This can be converted to the following binary constraint

$$y^e_{(q_{i_j},q_{i_j+1})} - y^S_{(q_j,q_{j+1})} \leq 0, \ \ i_j < j, \ \ \forall (q_{i_j},q_{i_j+1}) \in \mathcal{E}_{l,a}, \ \ (q_j,q_{j+1}) \in \mathcal{E}_{l,S}, \quad (41)$$

(3) There are transfer rules that cannot activate simultaneously. For example, either $T_{p,q}$ is true or $T_{q,p}$ is true. Both cannot happen simultaneously.

(4) For contracts that have multiple excess atomic propositions, once an excess atomic proposition is true, all excess atomic propositions on further levels must also be true.

Operational Rules

Operational rules can be modeled in the same framework as outlined for PSC, i.e., by defining atomic propositions representing the states of various operating lines and facilities and then forming logical expressions representing the rules. Finally, the resulting logical expression rules can be converted into binary constraints in exactly the same way as described earlier.

Planning Objectives

The objective functions considered in this model are operational objectives for the reasons discussed earlier. The model is a production-allocation model, therefore the optimal solution point (representing the production rates, flowrates and pressure distribution that satisfies all requirements, i.e., an operational policy) is of more interest than the optimal solution value. Following are three objectives that may be of interest from an operational perspective:

(1) The upstream operator is interested in maximizing the delivery of dry gas to the LNG plants. This is because the

more gas sold, the higher the revenue for the operator. Of course, the gas flowrates cannot go above the maximum demand rates set by contracts. Mathematically this is a minimization since demand nodes are sinks and hence $Q_{s,i}$ is nonpositive. This objective function is therefore bounded above by 0 and below by the negative of the sum of maximum demand rates $\sum_{i \in \mathcal{N}_D} -\Lambda^U_{d,i}$.

$$z_g = \sum_{i \in \mathcal{N}_D} Q_{s,i} \tag{42}$$

(2) NGL sharing is not governed by contracts. Instead NGL are shared according to the ownership of the fields. Hence, if two production strategies produce the same amount of dry gas but different amounts of condensates, the one with higher condensate production is preferred since the upstream operator receives more condensate and hence a higher revenue.

$$z_L = -\sum_{i \in \mathcal{F}_w} Q_{Ls,i} \tag{43}$$

(3) It may be of interest to the upstream operator to prioritize production from certain fields. The motivation for doing so may come from long range production-planning models or reservoir-management models, that may dictate the long-term production profile for a particular field. Over a short term, the interpretation of these profiles may be to prioritize certain fields. The simplest model for prioritizing production is to simply maximize the production from these fields. Define a set $\mathcal{F}_{pr} \subset \mathcal{F}$ that is the set of fields that should have high priority. Then this objective is represented as:

$$z_{pr} = -\sum_{i \in \mathcal{F}_{pr}} Q_{s,i} \tag{44}$$

It should be noted that equalities 42, 43 and 44 are represented as constraints in the model to facilitate calculation of all the three quantities when using one of them as objective. The actual objective function is given by:

$$\begin{aligned} &\min z \\ &z = z_o, \end{aligned} \tag{45}$$

where subscript o is either g, L, or pr depending on the objective for the particular instance of the model. The domain of the optimization is not indicated explicitly to simplify notation, but it is over all the decision variables defined earlier.

The Case Study

We present a case study to demonstrate the modeling approach outlined earlier. This case study has all the features of the SGPS model and hence accurately represents an application of the modeling framework described earlier. However it is not a model of the SGPS for the reasons outlined below. This has been done to preserve the confidentiality of the original system parameters that are business sensitive.

(1) The parameters in the system including reservoir pressures, compositions, condensate ratios, well performance parameters, field production bounds, constants in flowrate-pressure relationships and maximum demand rates have been altered from their values in the SGPS model. Hence the flowrate-pressure distribution in the infrastructure model is totally different from the SGPS model and does not relate to it in any way.

(2) Although the trunkline network used in the case study is the same as the SGPS, the facilities have been moved around due to changes in the parameters.

(3) The contractual model has been altered. However, it is of the same complexity as the SGPS contract model. The production-sharing contracts are as in Figure 3 and the complete set of rules can be found in the Appendix and in supplementary material. The customer requirements have been altered and are as in Table 2.

Information on subsets of nodes and arcs in the infrastructure and the contract model can be found in the supplementary material to the paper. The supplementary material also lists additional model parameter values for the case study.

Estimation of bounds

The importance of estimating the tightest possible bounds for the decision variables in a nonconvex optimization problem is well known. The most important variable bounds on the system are bounds on the demand rates at the LNG plants, the pressures at the slugcatchers and the dry gas production rate from fields.

This problem has been observed to be especially sensitive to the bounds on the production rate. The optimal solution point is strongly influenced by the bounds set for production rates and pressures. A completely different solution point with the same optimal solution value can be found by varying the bounds. Roughly, the flowrate and pressure distribution in the network is driven by the bounds set on the field variables while the optimal solution value is dependent on the bounds on the demand rates and delivery pressures. Bounds are therefore as important as model parameters in this problem. The supplementary material contains an exhaustive discussion of the bounds.

Solution approach

The final model is a MINLP with nonconvex constraints. The model is formulated in GAMS.[49] It has a total of 827 variables with 804 continuous variables and 23 binary variables (reported using GAMS CONVERT by converting the model from a set based GAMS representation to a scalar GAMS representation). There are a total of 1086 constraints with 702 equalities (of which 220 are nonlinear) and 384 inequalities.

The model is solved with a branch-and-reduce algorithm[50,51] as implemented in BARON 7.5[52] with GAMS 22.2 (64-bit version). The CPU times are as reported by BARON on a 3.2 GHz Xeon dual processor machine running Linux kernel. SNOPT[53] was used as the NLP solver and CPLEX[54] was used as an LP solver for BARON. The constraint satisfaction tolerance was 10^{-6}.

All variables in the model are in the same units as presented in the paper with the exception of NGL rates (that are scaled by a factor of 100 from the units in the paper). It must be noted that this scaling introduces multiplying factors for the parameters in the constraints involving these variables. Also, all composition units in the model are in mole fractions although the field compositions in supplementary material and quality specifications in Table 2 are in mole percentages.

Table 3. Dry Gas Maximization Objective

Case	Dry Gas (hm^3/d)	NGL (m^3/d)	Priority Gas (hm^3/d)	Best Possible (hm^3/d)	Time, CPUs
(1) No gas quality, PSC and operational rules	109	22,781	48	110	237
(2) No PSC and operational rules	102	24,116	34	103	205
(3) Full model (Base Case)	94	21,440	30	95	19,237

Dry gas maximization

To elucidate the characteristics of the model and the solution, it is instructive to solve the following three cases with dry gas maximization (i.e., minimization of z_g). The first case excludes the PSC model and the gas quality specifications and is a solution of the infrastructure model with the rate and pressure constraints at demand nodes. The second case is the infrastructure model with gas quality constraints but excluding the PSC and operational rules. Finally, the third case is the entire model as presented.

All three cases are a MINLP. They are solved with a termination criterion of 1% relative gap between upper and lower estimates on the solution value. The base case (case (3)) was initialized with the solution from case (2). The lower bound on z_g was set equal to the lower bound on the solution value $z_g^{L,2}$ at termination (and therefore satisfying the termination criterion) obtained by solving case (2).

$$z_g^L = z_g^{L,2} = -102.934959524$$

This is quite important since in the absence of this bound, the solution procedure fails to converge even in 26+ h.

It should be noted that the solution times are not directly comparable since all three cases have different numbers of variables, both binary and continuous, as well as different numbers of constraints. Objective values and solution times for the three cases are compared in Table 3. "Best possible" column in the table presents the lower bound on the objective value at termination (of the corresponding minimization problem, therefore the best-possible value bounds the actual production value from above).

The detailed solution for case (3) (i.e., the base case) is presented in the supplementary material. The following general features of the problem can be noted:

(1) PSC add significant complexity to the problem as is evident from the fact that time required for solution of case (3) is two orders of magnitude greater than the cases (1) and (2).

(2) The binary relaxation of the base case (which is a nonconvex NLP) can be solved with SNOPT in less than 1 second. Also, the relaxed solution value (97 hm^3/d) turns out to be a valid lower bound and is tight with respect to the base case solution value (94 hm^3/d). It must be noted that because the NLP is noconvex, there is no guarantee that the relaxed solution value will underestimate the global solution value of the original MINLP.

(3) The branch-and-bound on the base case behaves contrary to the usual behavior observed in nonconvex NLP and MINLP. The conventional wisdom is that branch-and-bound usually locates the global solution in the first 10–20% of solution time and rest of the time is spent verifying that it is indeed the solution. However, this problem can display behavior in certain cases when till the very end of the solution procedure (>90% of the solution time) there is a wide gap between upper and lower bound with either both bounds away from the global solution value or lower bound is actually closer to the global solution value. A plausible explanation of this behavior may be that a small subset of the hyperrectangle defined by bounds is feasible for certain values of the parameters and therefore unless a partition containing the actual solution gets small enough, the bound fail to converge to the global solution value. This implies that even a multi-start strategy employing a local solver may fail to solve the problem if one of the initial guess is not close enough to feasible region and therefore, global optimization algorithms are indispensable to solve this problem.

(4) There are multiple globally optimal solutions to the problem with the same objective value due to the network structure of the problem where it is possible to deliver the same amount at the demand nodes with several different pressure, flowrate and production rate distributions in the network. This presents a problem in terms of choosing an operational state for the system. Some operational states may be more favorable than others (to a human operator) because of factors that may have not been represented in the model.

(5) There is a dilemma between setting tight bounds and loose bounds. We need a solution implementable on the network, however we do not want to restrict the feasible set artificially. Also too loose bounds create problems for the convergence.

(6) The time required for solution can be very different even with a slight change in the parameters and the bounds.

The following are specific characteristics of the solution:

(1) *Effect of Quality Constraint*: Between cases (1) and (2), a major redistribution of production rates takes place to satisfy quality. Quality constraints force the production rates from fields with high CO_2 and H_2S concentrations, i.e., B11, M3 and F6 to decrease. Also GHV constraints and composition constraints on C_4 and C_{5+} force production from HL and M3 to drop since they contain less C_1 and high levels of C_4 and C_{5+}. On the other hand, production from fields such as F23, F23SW and BN increase, since they have high levels of C_1, which compensate for the drop from the other fields, although not fully since the overall production rate drops by 7 hm^3/d, Although the difference between the objective values of case (1) and (2) is only 7 hm^3/d, the solutions are actually very different and bear no resemblance to each other. A substantial drop occurs in LNG 3 delivery because the production from fields physically connected to LNG 3 drops.

(2) *Effect of PSC Rules*: The incorporation of PSC rules results in further cuts in production from M1 and JN, which is only partially compensated by a rise in SC and M4 production rates and hence a net decrease in the delivery is observed. This decrease is also reflected by contracts B and C being in deficit. A redistribution of delivery between the LNG plants takes place due to PSC rules. In particular, there

Table 4. Various Objective Runs

Objective	Dry Gas (hm^3/d)	NGL (m^3/d)	Priority Gas (hm^3/d)	Best Possible	Time, CPUs
Dry Gas Maximization	92 (94*)	20,450	30	102	41,424
NGL Maximization	91	22,510	31	23,809	24,238
Priority Maximization	93	21,700	32	35	16,042

*1% gap, 19,237 CPUs.

is a major decrease in contract B production and therefore LNG 2 supply. This leads to excess contract A supply being freed up to supply LNG 3 via contract C and D and leads to an increase in LNG 3 delivery.

(3) Supply from contract F is close to zero due to quality constraints being in force since the field producing under F is a high CO_2 field.

Comparison of different objectives

The full problem can also solved with the other objectives described earlier as demonstrated in Table 4. All these runs were started without any initial guess to test if convergence is possible in absence of a sensible initial guess. However, it is possible to use an initial guess from the dry gas maximization case for the other solutions since this solution is feasible for all other runs. It is also possible to add bounds to the objective variable as described in the previous section that can significantly accelerate the convergence. The NGL and priority field objectives fail to converge even after 24 h with a 1% relative termination criterion. Hence, all the runs are solved with 10% relative termination criterion although as presented in previous section, dry gas maximization can be solved with a 1% relative termination criterion.

M4 and E11 production rates decrease in the NGL maximization case, while F6, SE, SC, and M1 production rates increase since fields with high condensate-gas ratios are favored in this solution. BN and HL production rates decrease as well because they do not contribute to NGL production. The delivery rate to LNG 2 decreases substantially while delivery to LNG 1 increases. This is because contract C is in deficit due to a decrease in M4 production, forcing contract A to transfer gas to C and therefore less gas is available in contract A for transfer to contract B, decreasing LNG 2 rates.

The priority fields in the priority maximization run have been chosen to be the high contaminant (sour) fields (B11, F6, E11, M1, and M4) because the upstream operator may be interested in depleting all fields equally. The gas quality constraints favor higher production rates from sweet fields and this can lead to these fields getting over-exploited and in the future there may not be enough sweet gas to blend the sour gas, and it may not be feasible to satisfy all the quality constraints. Maximizing production rates from the sour fields pushes the operation to the limit where any further increase in production rates from these fields will violate a quality constraint and therefore, is the maximum production rate possible from these fields. However, not all priority production rates increase in the priority solution because of the quality specifications. Indeed, E11, M4, and B11 register small decreases from the base case, however the F6 and M1 production rate increases overcompensate this decrease resulting in a net increase in the objective. Delivery to LNG 3 increases mostly due to inter-contract transfers from contract A since contract A has excess gas available due to the decrease in LNG 1 demand rate as well as from an increase in F6 supply. It should be noted that priority maximization yields a higher dry gas delivery than the base case because of the 10% relative termination criterion employed.

Hierarchical Multi-Objective Case Study

As aforementioned, there are multiple globally optimal solutions to the problem. Moreover, there are multiple objectives for the operation of the system. This feature of the problem can be leveraged to obtain a solution that maximizes several objectives in a hierarchical way.[55] Obtaining a Pareto-optimal solution for this problem is unnecessary (as well as prohibitively expensive due to nonconvexity) because there is a clear hierarchy of objectives as follows:

(1) The first priority is to supply the required amount of dry gas (represented by the maximum demand rates at the LNG plants), since this is mandated by PSC. Therefore maximization of the dry gas production rate has the highest priority.

(2) The second priority is to maximize the NGL production rate because it is beneficial for the upstream operator from a revenue perspective.

(3) Finally the last priority is to maximize the production rate from priority fields.

This hierarchical multi-objective optimization is done as follows:

(1) First, the MINLP is solved with the first objective, i.e., maximizing the dry gas production rate (minimizing z_g. Let the optimal solution value be z_g^{opt} (this is same as the base case).

(2) Next the upper bound to z_g is set to z_g^{opt}

$$z_g^U = z_g^{\text{opt}}$$

Also upper bounds are added to the other objectives (z_L and z_{pr}) to get a better value than this solution (z_L^{p1} and z_{pr}^{p1} respectively). This also helps to accelerate convergence.

$$z_L^U = z_L^{p1},$$
$$z_{\text{pr}}^U = z_{\text{pr}}^{p1}.$$

The MINLP is initialized with the previous solution point and is solved with the second objective, i.e., to maximize the NGL production rate (minimize z_L).

(3) The upper bounds of z_g and z_L are set at z_g^{opt} and at z_L^{opt}, respectively, i.e., the values of these variables in the solution of the second instance:

$$z_g^U = z_g^{\text{opt}},$$
$$z_L^U = z_L^{\text{opt}}.$$

Table 5. Hierarchical Multi-Objective Case Study

Objective	DryGas (hm^3/d)	Condensate (m^3/d)	Priority Gas (hm^3/d)	Best Possible	Time (s)
Base Case	92 (94*)	20,450	30	102	41,424
NGL Maximization	92	20,980 (22,476†)	30	23,315	86
Priority Maximization	92	22,020	32	34	4,717

*1% gap, 19237 CPUs.
†1% gap with a restart using objective cut as outlined in the section on ''dry gas maximization'' solution and initial guess solution with 10% gap, 1694 CPUs.

Similarly the upper bound of z_{pr} is fixed at the value of this variable z_{pr}^{p2} from this solution

$$z_{pr}^{U} = z_{pr}^{p2}.$$

The MINLP is initialized with the solution point from second problem and is solved to minimize z_{pr}.

It should be noted that the solution obtained at each step is very different in terms of the pressure-flowrate distribution in the network.

All runs are solved with a 10% relative termination criterion because although the dry gas maximization step can be solved with 1% relative termination criterion, the NGL maximization fails to converge in 24 h with a 1% relative termination criterion. A summary of the results is presented in Table 5.

The total NGL production rate after the second run is roughly 2.5% higher than the base case NGL production while maintaining the same production rate for dry gas. However, the production rate distribution in this solution is almost the same as with the dry gas maximization. This is because the NGL production in the dry gas maximization solution is very close to being within 10% of the bounds and a minor change satisfies the convergence criteria. Since the gap is big enough, a second case with the NGL maximization objective was run with a 1% relative termination criterion and an initial guess and objective variable bounds from this solution. This offers an improvement of 10% in the solution value. This improvement is substantial in financial terms to the upstream operator in comparison to the base case solution. The priority field solution value shows an increase of 10% over the base case, increasing their share from 32 to 35% of total production.

This approach is a powerful tool for planning operations in the system with multiple operational objectives and criteria, and in the presence of multiple optimal solutions. Customized solution approaches that exploit the problem structure and avoid redundant computation for the multi-objective case study may be required to solve the problem efficiently.

Conclusions and Future Work

A model for production allocation in the upstream natural gas supply chain has been presented with the SGPS as the primary case study. The model includes realistic pressure-flowrate relationships and represents multiple qualities of gas in the trunkline network. A framework for modeling the complicated production-sharing rules central to operation of the system is described.

Although the model has been developed with the SGPS in mind, it is general enough to handle most upstream gas and oil production systems including the complicated production-sharing rules invariably involved with such systems.

It is necessary to exploit problem structure to accelerate the convergence of the algorithms as well as to ensure reliability. It seems possible to exploit the unique features of the problem, e.g., a subset of the constraints are network constraints, the nature of contracts and so on, the physics of the flow in the network and that part of the problem is similar to the pooling problem, to customize the heuristics for algorithms and add additional valid constraints during iterations for a faster and more reliable solution procedure. This is expected to be a major area of investigation in the near future.

The following model improvements and extensions are also possible. The well-performance model can be improved by using a better response surface based on reservoir simulations. More detailed modeling of well and riser platforms would help to provide better control decisions for the system. Moreover, it can potentially also allow for preferential routing of gas in the network and would help to relax the perfect mixing assumption. A simplified model of the LPG plant in the model would help to incorporate the objective of maximizing LPG production that is of interest to the upstream operator. A representation of the LNG plant in the model can help the LNG plants to respond to upstream fluctuations. Although the last improvement is not directly applicable to the SGPS, as the LNG plants are owned by a third party, it is true for several other gas production systems where the upstream operator also owns the liquefaction facility. Finally, an economic representation of the system could be built on top of the model presented here where the contractual modeling framework is extended to include complex commercial and economic rules.

Acknowledgments

The authors would like to thank Dr. Alexander Mitsos who provided a lot of useful suggestions and insights, both in the development of the model and writing of this article. They also express gratitude to Andrew Hooks and K.L. Tan of Sarawak Shell Berhad (SSB), Malaysia for additional inputs on the SGPS operations. They would like to thank the management of SSB and Shell International Exploration and Production (SIEP), The Netherlands for the permission to use the SGPS as the case study. Finally, they would like to express gratitude to SIEP for funding this work.

Literature Cited

1. International energy outlook 2006. Energy Information Administration, U. S. Department of Energy, 2006.
2. Garvin WW, Crandall HW, John JB, Spellman RA. Application of linear programming in the oil industry. *Manag Sci*. 1957;3:407–430.
3. Dougherty EL. Application of optimization methods to oilfield problems: proved, probable, possible. In the 47th Annual Fall Meeting of SPE, San Antonio, Texas: Society of Petroleum Engineers, 1972: SPE 3978.
4. Durrer EJ, Slater GE. Optimization of petroleum and natural gas production—a survey. *Manag Sci*. 1977;24:35–43.
5. Huppler JD. Scheduling gas field production for maximum profit. *Soc Petrol Eng J*. 1974;14:279–294.
6. Flanigan O. Constrained derivatives in natural gas pipeline system optimization. *J Petrol Technol*. 1972;24:549–556.
7. O'Dell PM, Steubing NW, Gray JW. Optimization of gas field operations. *J Petrol Technol*. 1973;25:419–425.

8. Murray JE, III, Edgar TF. Optimal scheduling of production and compression in gas-fields. *J Petrol Technol*. 1978;30:109–116.
9. McFarland JW, Lasdon L, Loose V. Development planning and management of petroleum reservoirs using tank models and nonlinear programming. *Oper Res*. 1984;32:270–289.
10. Nygreen B, Christiansen M, Haugen K, Bjørkvoll T, Kristiansen Ø. Modeling Norwegian petroleum production and transportation. *Annl Oper Res*. 1998;82:251–267.
11. van den Heever SA, Grossmann IE, Vasantharajan S, Edwards K. A Lagrangean decomposition heuristic for the design and planning of offshore hydrocarbon field infrastructures with complex economic objectives. *Ind Eng Chem Res*. 2001;40:2857–2875.
12. Gao W, Hartley P, Sickles RC. Optimal dynamic production policy: the case of a large oil field in Saudi Arabia. Technical report, James A. Baker III Institute for Public Policy, Rice University, 2004.
13. Ayda-zade KR, Bagirov AG. On the problem of spacing of oil wells and control of their production rates. *Autom Rem Contr*. 2006; 67:44–53.
14. Haugen KK. A stochastic dynamic programming model for scheduling of offshore petroleum fields with resource uncertainty. *Eur J Oper Res*. 1996;88:88–100.
15. Goel V, Grossmann IE, El-Bakry AS, Mulkay EL. A novel branch and bound algorithm for optimal development of gas fields under uncertainty in reserves. *Comput Chem Eng*. 2006;30:1076–1092.
16. Yusgiantoro P, Hsiao FST. Production-sharing contracts and decision-making in oil production—the case of Indonesia. *Energy Econ*. 1993;15:245–256.
17. Marqués D, Morari M. On-line optimization of gas pipeline networks. *Automatica*. 1988;24:455–469.
18. Osiadacz AJ, Bell DJ. A simplified algorithm for optimization of large-scale gas networks. *Opt Contr Appl Methods*. 1986;7:95–104.
19. Furey BP. A sequential quadratic programming-based algorithm for optimization of gas networks. *Automatica*. 1993;29:1439–1450.
20. De Wolf D, Smeers Y. Optimal dimensioning of pipe networks with application to gas transmission networks. *Oper Res*. 1996;44:596–608.
21. De Wolf D, Smeers Y. The gas transmission problem solved by an extension of the simplex algorithm. *Manag Sci*. 2000;46:1454–1465.
22. Dahl HJ, Rømo F, Tomasgard A. Optimisation model for rationing-efficient allocation of capacity in a natural gas transportation network. In: Conference proceedings of the International Association for Energy Economics, Prague, 2003.
23. Ríos-Mercado RZ, Wu S, Scott LR, Boyd EA. A reduction technique for natural gas transmission network optimization problems. *Annl Oper Res*. 2002;117:217–234.
24. Martin A, Möller M, Moritz S. Mixed integer models for the stationary case of gas network optimization. *Math Program*. 2006; 105:563–582.
25. Ríos-Mercado RZ, Kim S, Boyd EA. Efficient operation of natural gas transmission systems: a network-based heuristic for cyclic structures. *Comput Oper Res*. 2006;33:2323–2351.
26. Wattenbarger RA. Maximizing seasonal withdrawals from gas storage reservoirs. *J Petrol Technol*. 1970;22:994–998.
27. O'Neill RP, Williard M, Wilkins B, Pike R. A mathematical programming model for allocation of natural gas. *Oper Res*. 1979; 27:857–873.
28. Levary RR, Dean BV. A natural gas flow model under uncertainty in demand. *Oper Res*. 1980;28:1360–1374.
29. Brooks RE. Using generalized networks to forecast natural gas distribution and allocation during periods of shortage. *Math Program Study* 1981;15:23–42.
30. Guldmann J. Supply, storage and service reliability decisions by gas distribution utilities: a chance constrained approach. *Manag Sci*. 1983;29:884–906.
31. Avery W, Brown GG, Rosenkranz JA, Wood RK. Optimization of purchase, storage and transmission contracts for natural gas utilities. *Oper Res*. 1992;40:446–462.
32. Contesse L, Ferrer JC, Maturana S. A mixed-integer programming model for gas purchase and transportation. *Annl Oper Res*. 2005; 139:39–63.
33. Gabriel SA, Kiet S, Zhuang J. A mixed complementarity-based equilibrium model of natural gas markets. *Oper Res*. 2006;53:799–818.
34. Kosmidis VD, Perkins JD, Pistikopoulos EN. Optimization of well oil rate allocations in petroleum fields. *Ind Eng Chem Res*. 2004;43:3513–3527.
35. Ortíz-Gómez A, Rico-Ramirez V, Hernández-Castro S. Mixed-integer multiperiod model for the planning of oilfield production. *Comput Chem Eng*. 2002;26:703–714.
36. Barragán-Hernández V, Vázquez-Román R, Rosales-Marines L, García-Sánchez F. A strategy for simulation and optimization of gas and oil production. *Comput Chem Eng*. 2005;30:215–227.
37. Tomasgard A, Rømo F, Fodstad M, Midthun K. Optimization models for the natural gas value chain. Technical report, Norwegian University of Science and Technology, and SINTEF, Trondheim, Norway, 2005.
38. Model documentation: Natural gas transmission and distribution module of the National Energy Modeling System. Energy Information Administration, U. S. Department of Energy, 2006; DOE/EIA-M062.
39. Bertsekas DP. *Nonlinear Programming,* 2nd ed. Belmont, MA: Athena Scientific, 1999.
40. Bazaraa MS, Sherali HD, Shetty CM. *Nonlinear Programming: Theory and Algorithms*, 2nd ed. Hoboken, NJ: Wiley, 1993.
41. Grossmann IE. Review of nonlinear mixed-integer and disjunctive programming techniques. *Opt Eng*. 2002;3:227–252.
42. Katz DL, Lee RL. *Natural Gas Engineering: Production and Storage*. New York: McGraw-Hill, 1990.
43. Glover F. Improved linear integer programming formulations of nonlinear integer problems. *Manag Sci*. 1975;22:455–460.
44. Haverly CA. Studies of the behavior of recursion for the pooling problem. *ACM SIGMAP Bull*. 1978;25:19–28.
45. Haverly CA. Behavior of recursion model—more studies. *ACM SIGMAP Bull*. 1979;26:22–28.
46. ISO6976:1995. Natural gas—calculation of calorific values, density, relative density and Wobbe index from composition. *Technical report, International Organization for Standardization*, 1995; p. 12, Table 4.
47. Post S. Reasoning with incomplete and uncertain knowledge as an integer linear program. In: Proceeding Avignon 1987: Expert Systems and Their Applications. Avignon, France. 1987;2:1361–1377.
48. Raman R, Grossmann IE. Relation between MILP modelling and logical inference for chemical process synthesis. *Comput Chem Eng*. 1991;15:73–84.
49. General algebraic and modeling system. version 22.2. http://www.gams.com.
50. Tawarmalani M, Sahinidis NV. *Convexification and Global Optimization in Continuous and Mixed-Integer Nonlinear Programming: Theory, Algorithms, Software and Applications*. Dordrecht, The Netherlands: Kluwer, 2002.
51. Tawarmalani M, Sahinidis NV. Global optimization of mixed-integer nonlinear programs: a theoretical and computational study. *Math Program*. 2004;99:563–591.
52. Sahinidis NV, Tawarmalani M. BARON 7.5: Global optimization of mixed-integer nonlinear programs, User's Manual, Available at: http://www.gams.com/dd/docs/solvers/baron.pdf, 2005.
53. Gill PE, Murray W, Saunders MA. SNOPT: an SQP algorithm for large-scale constrained optimization. *SIAM Rev*. 2005;47:99–131.
54. CPLEX. Version 10.0. http://ilog.com/products/cplex.
55. Ahmad BS, Barton PI. Process-wide integration of solvent mixtures. *Comput Chem Eng*. 1999;23:1365–1380.

Appendix

Contractual rules

In this section, the production-sharing framework for the case study is described in brief. A more detailed version of this section appears in the supplementary material to the article.

Contract Definitions. There are five contracts in the system. The field contract assignment is provided in Table A1. Following are the rules dictating contract plant assignments:

(1) LNG plant 1 must be supplied by contract A, X and Y. There are no special rules for separating them, so we combine supplies from A, X, and Y and denote it by contract A.

(2) LNG plant 2 must be supplied by contract B.

(3) LNG plant 3 must be supplied in the ratio 600:350 by contract C and D. Hence the following constraint is enforced for flowrates in the contract graph:

$$350q_{a,(C_0,CD_d)} = 600q_{a,(D_0,CD_d)}. \tag{46}$$

(4) Contract F should normally supply contract B (i.e., LNG plant 2), however under certain conditions, it may supply contract A (explained later).

Inter-Contract Transfer Rules. This section describes briefly the inter-contract transfer rules, inferences of these rules and their logical representation in the model. Their CNF and binary constraint representation can be found in the supplementary material. The priorities for supply when a contract is in excess are defined in Table A2. The rules are as follows:

(1) When demand of LNG plant 2 cannot be fulfilled by contract B, then gas shall be borrowed from A to supply plant 2. RA to RB shall be open at this stage.

$$\begin{aligned}(E_{(A_0,A_1)} \wedge \neg E_{(B_1,B_2)}) &\Rightarrow (T_{A,B} \wedge C_{(RA,RB)}).\\ T_{A,B} &\Rightarrow C_{(RA,RB)}\\ \neg(E_{(A_0,A_1)} \wedge \neg E_{(B_1,B_2)}) &\Rightarrow \neg T_{A,B},\end{aligned}$$

(2) If demand of LNG plant 2 cannot be fulfilled by contract B and A is unable to fulfill this deficit, then gas shall be borrowed from contract C to supply plant 2

$$\begin{aligned}(\neg E_{(B_1,B_2)} \wedge E_{(C_0,C_1)} \wedge S_{(D_1,D_2)}) &\Rightarrow T_{C,B}.\\ \neg(\neg E_{(B_1,B_2)} \wedge E_{(C_1,C_2)} \wedge S_{(D_1,D_2)}) &\Rightarrow \neg T_{C,B}.\end{aligned}$$

(3) If contract D cannot meet its allocated production share, C shall have the first priority to fulfill this deficit, followed by contract B, then A.

$$\begin{aligned}(\neg E_{(D_0,D_1)} \wedge E_{(C_0,C_1)}) &\Rightarrow T_{C,D},\\ (\neg E_{(D_0,D_1)} \wedge E_{(B_1,B_2)} \wedge S_{(C_2,C_3)}) &\Rightarrow T_{B,D},\\ (\neg E_{(D_0,D_1)} \wedge E_{(A_0,A_1)} \wedge S_{(B_2,B_3)}) &\Rightarrow T_{A,D}.\\ \neg(\neg E_{(D_0,D_1)} \wedge E_{(C_0,C_1)}) &\Rightarrow \neg T_{C,D},\\ \neg(\neg E_{(D_0,D_1)} \wedge E_{(B_1,B_2)} \wedge S_{(C_2,C_3)}) &\Rightarrow \neg T_{B,D},\\ \neg(\neg E_{(D_0,D_1)} \wedge E_{(A_0,A_1)} \wedge S_{(B_2,B_3)}) &\Rightarrow \neg T_{A,D}.\end{aligned}$$

(4) If contract C cannot meet its allocated production share, D shall have the first priority to fulfill this deficit, followed by contract B, then A.

Table A1. Field Contract Assignments

$i \in \mathcal{C}$	$j \in \mathcal{C}_i^S$	Fields $\mathcal{F}_j$
A	A	SC, F6, F23
	X	F23SW
	Y	BN,BY,D35
B	B	M1, M3, B11
C	C	M4, HL, JN
D	D	SE
F	F	E11

Table A2. Priority of Supply

Supplying Contract	Priority of Receiving if in Deficit
A	B, C, D
B	C, D
C	D, B
D	C
F	B, A

$$\begin{aligned}(\neg E_{(C_0,C_1)} \wedge E_{(D_0,D_1)}) &\Rightarrow T_{D,C},\\ (\neg E_{(C_0,C_1)} \wedge E_{(B_1,B_2)}) &\Rightarrow T_{B,C},\\ (\neg E_{(C_0,C_1)} \wedge E_{(A_0,A_1)} \wedge S_{(B_2,B_3)}) &\Rightarrow T_{A,C}.\\ \neg(\neg E_{(C_0,C_1)} \wedge E_{(D_0,D_1)}) &\Rightarrow \neg T_{D,C},\\ \neg(\neg E_{(C_0,C_1)} \wedge E_{(B_1,B_2)}) &\Rightarrow \neg T_{B,C},\\ \neg(\neg E_{(C_0,C_1)} \wedge E_{(A_0,A_1)} \wedge S_{(B_2,B_3)}) &\Rightarrow \neg T_{A,C}.\end{aligned}$$

(5) F supply normally belongs to contract B. However if B is in excess and A is in deficit, F may supply A.

$$\begin{aligned}(\neg E_{(A_0,A_1)} \wedge (E_{(B_0,B_1)} \wedge E_{(B_1,B_2)})) &\Rightarrow T_{F,A},\\ \neg E_{(B_0,B_1)} &\Rightarrow T_{F,B},\\ \neg E_{(B_1,B_2)} &\Rightarrow \neg T_{F,A},\\ E_{(A_0,A_1)} &\Rightarrow \neg T_{F,A},\\ T_{F,B} &\Rightarrow C_{(RA,RB)}.\end{aligned}$$

Following are the additional logical rules added to the problem:

(1) Only one of the following two transfers should activate: $T_{C,D}$ and $T_{D,C}$,

$$\neg(T_{D,C} \wedge T_{C,D}).$$

(2) Contract B has two excess atomic propositions $E_{(B_0,B_1)}$ and $E_{(B_1,B_2)}$. If the contract is already in excess at level 0, it must be excess at further levels. Logically this can be represented as

$$E_{(B_0,B_1)} \Rightarrow E_{(B_1,B_2)}.$$

Operational Rules. *(1) A minimum flow rate of 14.158 hm³/d (500 MMscfd) shall be maintained in the (M1,T) line. In the event that M1 production is less than 14.158 hm³/d, part of JN production shall be diverted into this pipeline section.**

$$\begin{aligned}C_{M1} &\Rightarrow C_{JN,(M1,RC)},\\ \neg C_{M1} &\vee C_{JN,(M1,RC)}.\end{aligned}$$

(2) Processing capacity at M1 platform is 36.812 hm³/d (1300 MMscfd) of which 21.238 hm³/d (750 MMscfd) capacity belongs to JN production.

Manuscript received Mar. 16, 2007, and revision received Oct. 22, 2007.

* C_{M1} and $C_{JN,(M1,RC)}$ are atomic propositions that represent "M1 production is greater than 14.158 hm³/d" and "all of JN production is diverted into (M1,RC)" respectively.